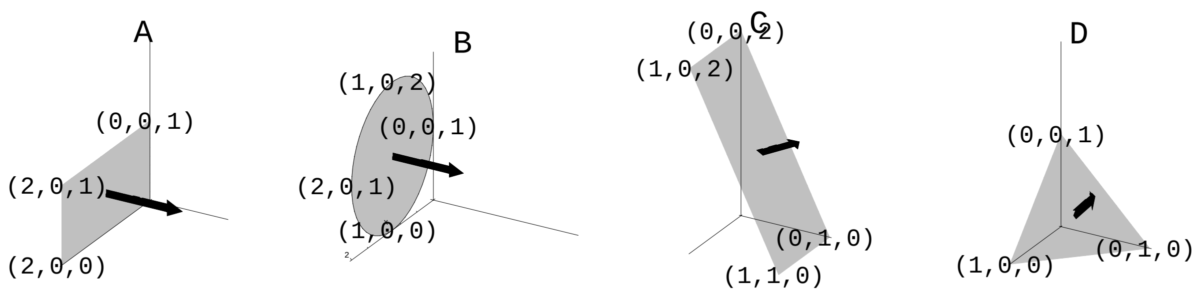
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find curl $\mathbf{F}$ and find div $\mathbf{G}$, if the vector field $\mathbf{F} = \langle xy^2z^3, 0, x^3y^2z \rangle$ and if the vector field $\mathbf{G} = \langle z\sin(x/y), z\sin(x/y), z\sin(x/y) \rangle$.

2. Find a scalar field f so that $\nabla f = \mathbf{F} = \langle 2x+y+z, x+3y^2+z, x+y \rangle$ and use it to compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ if $\mathbf{r}(t)$ is an extremely complex helical spiral that goes from $(1,1,1)$ to $(5,-1,2)$

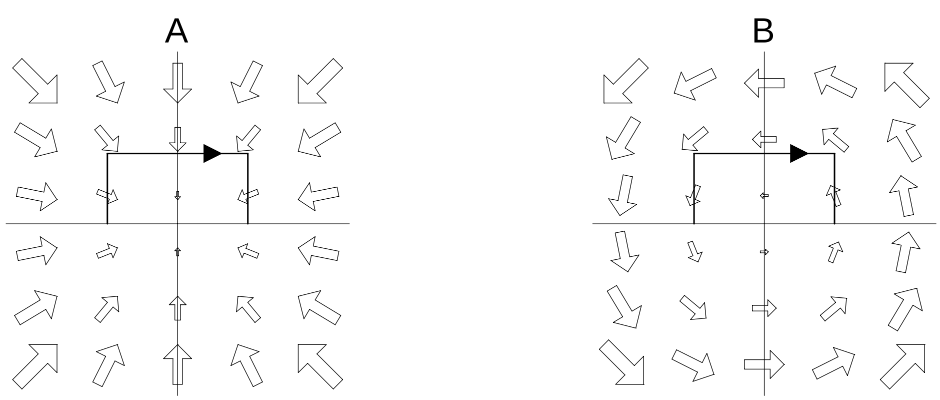
3. Vector, scalar or nonsense. $f = f(x,y,z)$ and $g = g(x,y,z)$ are scalar fields and $\mathbf{F} = \mathbf{F}(x,y,z)$ and $\mathbf{G} = \mathbf{G}(x,y,z)$ are vector fields. Determine if the given object is a scalar field, a vector field or nonsense.
A. $\mathbf{F}\times\mathbf{G}$ B. curl f C. $\mathbf{F}\cdot\mathbf{G}$ D. $f\mathbf{F}$ E. div(grad f) F. grad(div $\mathbf{F}$) G. div(curl $\mathbf{G}$)
H. grad $\mathbf{F}$ I. $(\mathbf{F}\cdot\mathbf{G})\mathbf{F}$ J. (grad f) $\cdot$ (grad g)

4. Write down a triple integral which will give mass of the the region in 3-space that is in the first octant and under the surface $x+2y+3z=6$ with density function $\delta(x,y,z) = xy^3z^2$. Do **NOT** evaluate.



5. The vector field $\mathbf{F} = \langle 2,5,3 \rangle$ compute the flux of $\mathbf{F}$ over each of the surfaces above. Each can be done without evaluating an integral. [Hint B is a circle, D is an equilateral triangle, A and C are rectangles.]

6. Find formula's for the two vector fields below. (There are many possible answers). Decide if the line integrals over the given curve (three sides of a rectangle) will be positive, negative or zero for each vector field.



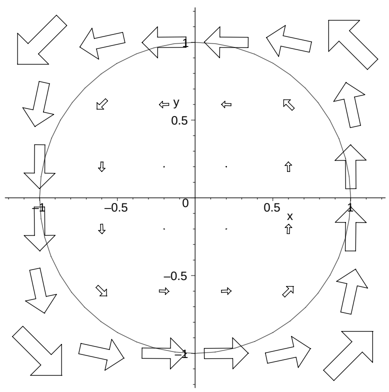
7. Let W be a region in 3-space with smooth boundary and let S be its boundary ∂W with outward normal. Show for the constant vector field $\mathbf{F} = \langle a_1, a_2, a_3 \rangle$, the flux integral $\iint_S \mathbf{F}\cdot d\mathbf{A} = 0$. [Hint: Use a Theorem.]

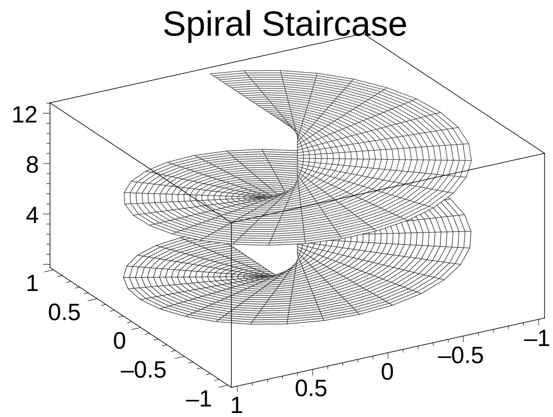
There is more test on the other side.

8. Sketch the region and rewrite the cylinderical triple integeral below in both spherical and rectangular coordinates, do **NOT** evaluate. (Here $\delta = \delta(x, y, z) = \delta(z, r, \theta) = \delta(\rho, \phi, \theta)$ is some density function.)

$$\int_0^{\pi} \int_0^4 \int_r^4 \delta \; r \; dz \; dr \; d\theta$$

9. Let C be the unit circle in the plane oriented counter-clockwise and let $\mathbf{F} = \langle -y^3, x^3 \rangle$ (below left). Carefully write and simplify the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ til it is a simple (non-vector) integral in the parameter t. Use your TI-89 to evaluate the integral. Use Green's Theorem to rewrite the line integral as a double integral over the region D of points where $x^2 + y^2 \leq 1$. Convert the double integral to polar coordinates and evaluate.





10. Compute the flux integral for $\mathbf{F} = \mathbf{k}$ over the parametic surface $\mathbf{r}(s, t) = \langle s \cos t, s \sin t, t \rangle$ for $0 \leq s \leq 1$ and $0 \leq t \leq 4\pi$ oriented upward (The spiral staircase, above right).