Show ALL work for credit; be neat; and use only ONE side of each page of paper. Do NOT write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. A sphere has its center at \((-1, 2, 3)\) and is tangent to the \(xz\)-plane, (the sphere ‘just touches’ this plane), find the equation of this sphere.

2. Find the equation of the plane through the points \((2, 1, -2), (3, -1, 2)\) and \((4, 0, 1)\).

3. Let \(P(3, -2, 2)\) and \(\vec{v} = \langle 3, -1, 5 \rangle\), find:
   A. The equation of the line through \(P\) in the direction of \(\vec{v}\).
   B. The coordinates of the point where the line in A intersects the \(xz\)-plane
   C. The equation of the plane perpendicular to \(\vec{v}\) through \(P\).
   D. The coordinates of the point where the \(y\)-axis intersects the plane in C.

4. The following are maple contour plots of \(z = 2x + y\), \(z = x + y\), \(z = x + 2y\), \(z = x + y^2\) and \(z = x^2 + y\) over the range \(x = -3..3\) and \(y = -3..3\). Match the plots to the functions.

5. Make sure your TI-89 is in radian mode and use it to do parts A & B.
   A. Find the exact answer to \(\int \sin^4 t \cos^3 t \, dt\).
   B. Plot the curve \(r(t) = (\cos^4 t, \sin^4 t)\) for \(0 \leq t \leq \pi/2\)
   C. Find the velocity \(\vec{v}(t)\) of the curve in B.
   D. Find the acceleration \(\vec{a}(t)\) of the curve in B.

6. A treasure map reads start at the big X, walk 40 paces north, 20 paces northwest and dig a hole 10 paces deep. Write the vector \(\vec{v}\) that goes from the big X to the bottom of the hole and find the exact simplified value of the length squared, \(||\vec{v}||^2\). (The x-axis points East, the y-axis points North, and the z-axis points up.)

7. Polar coordinates.
   A. Convert the polar equation \(r = 2 \cos \theta + 4 \sin \theta\) to rectangular (Cartesian) coordinates and COMPLETELY identify the curve and sketch it.
   B. Convert \(1 = x^2 y + y^3\) to polar coordinates, and solve for \(r\). Simplify. [hint: the answer is some trig function to some power.]

8. Using vector operations write \(\vec{a} = \langle 2, -1, 5 \rangle\) as the sum of two vectors, one parallel (say \(\vec{v}\)), and one perpendicular (say \(\vec{w}\)) to \(\vec{b} = (-4, 4, 2)\).

9. Find parametric equations of the line of intersection of the two planes \(x + 2y + 2z = 3\) and \(3x + 2y - 2z = 9\).

10. We want to determine the distance between the two skew lines below two different ways.
    \(L_1: x = 2 + t, y = 2 - t, z = 5 + 3t\) \hspace{1cm} \(L_2: x = 1 - s, y = 1 - s, z = 2s\)
    A. First find a vector \(\vec{n}\) that is perpendicular to both lines.
    B. Find the equation of the plane perpendicular to \(\vec{n}\) that contains \(L_1\) in the form \(ax + by + cz + d = 0\)
    C. To complete the first method, use the expression
        \[
        \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}
        \]
    D. Find the vector \(\vec{v}\) that goes from \(L_1\) at \(t = 0\) to \(L_2\) at \(s = 0\) and \(\hat{n}\) the unit vector in the \(\vec{n}\) direction.
    E. Use the scalar projection of \(\vec{v}\) in the \(\hat{n}\) direction to complete the second method.