MAC 2313 Calculus 3 **Test 1** 16 Sep 2003

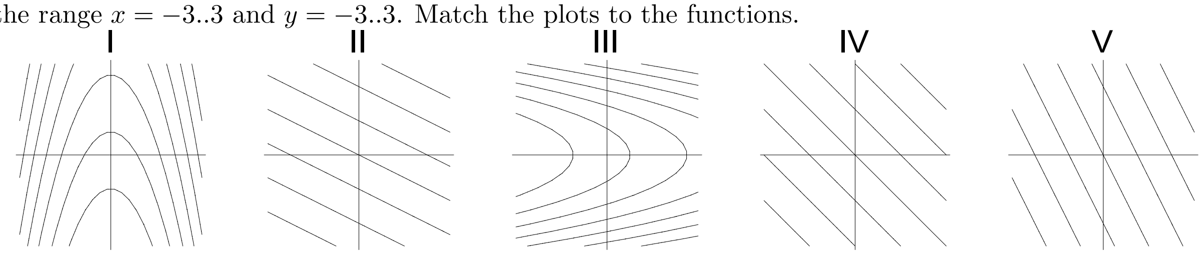
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. A sphere has its center at $(-1, 2, 3)$ and is tangent to the xz-plane, (the sphere 'just touches' this plane), find the equation of this sphere.

2. Find the equation of the plane through the points (2, 1, -2), (3, -1, 2) and (4, 0, 1).

3. Let $P(3, -2, 2)$ and $\vec{v} = \langle 3, -1, 5 \rangle$, find:
 A. The equation of the line through P in the direction of $\vec{v}$.
 B. The coordinates of the point where the line in A intesects the xz-plane
 C. The equation of the plane perpendicular to $\vec{v}$ through P.
 D. The coordinates of the point where the y-axis intersects the plane in C.

4. The following are maple contour plots of $z = 2x + y, z = x + y, z = x + 2y, z = x + y^2$ and $z = x^2 + y$ over the range $x = -3..3$ and $y = -3..3$. Match the plots to the functions.



5. Make sure your TI-89 is in radian mode and use it to do parts A & B.
 A. Find the exact answer to $\int \sin^4 t \cos^3 t \; dt$.
 B. Plot the curve $\mathbf{r}(t) = \left\langle \cos^4 t, \sin^4 t \right\rangle$ for $0 \le t \le \pi/2$
 C. Find the velocity $\mathbf{v}(t)$ of the curve in B.
 D. Find the acceleration $\mathbf{a}(t)$ of the curve in B.

6. A treasure map reads start at the big X, walk 40 paces north, 20 paces northwest and dig a hole 10 paces deep. Write the vector $\vec{v}$ that goes from the big X to the bottom of the hole and find the exact simplified value of the length squared, $||\vec{v}||^2$. (The x-axis points East, the y-axis points North, and the z-axis points up.)

7. Polar coordinates.
 A. Convert the polar equation $r = 2\cos\theta + 4\sin\theta$ to rectangular (Cartesian) coordinates and COMPLETELY identify the curve and sketch it.
 B. Convert $1 = x^2 y + y^3$ to polar coordinates, and solve for r. Simplify. [hint: the answer is some trig function to some power.]

8. Using vector operations write $\mathbf{a} = \langle 2, -1, 5 \rangle$ as the sum of two vectors, one parallel (say $\mathbf{v}$), and one perpendicular (say $\mathbf{w}$) to $\mathbf{b} = \langle -4, 4, 2 \rangle$.

9. Find parametric equations of the line of intersection of the two planes $x + 2y + 2z = 3$ and $3x + 2y - 2z = 9$.

10. We want to determine the distance between the two skew lines below two different ways.
L_1: $x = 2 + t, y = 2 - t, z = 5 + 3t$ $\qquad$ L_2: $x = 1 - s, y = 1 - s, z = 2s$
 A. First find a vector $\vec{n}$ that is perpendicular to both lines.
 B. Find the equation of the plane perpendicular to $\vec{n}$ that contains L_1 in the form $ax + by + cz + d = 0$
 C. To complete the first method, use the expression

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

 D. Find the vector $\vec{v}$ that goes from L_1 at $t = 0$ to L_2 at $s = 0$ and $\hat{n}$ the unit vector in the $\vec{n}$ direction.
 E. Use the scalar projection of $\vec{v}$ in the $\vec{n}$ direction to complete the second method.