MAC 2313 Calculus 3 **Test 2** 21 Oct 2003

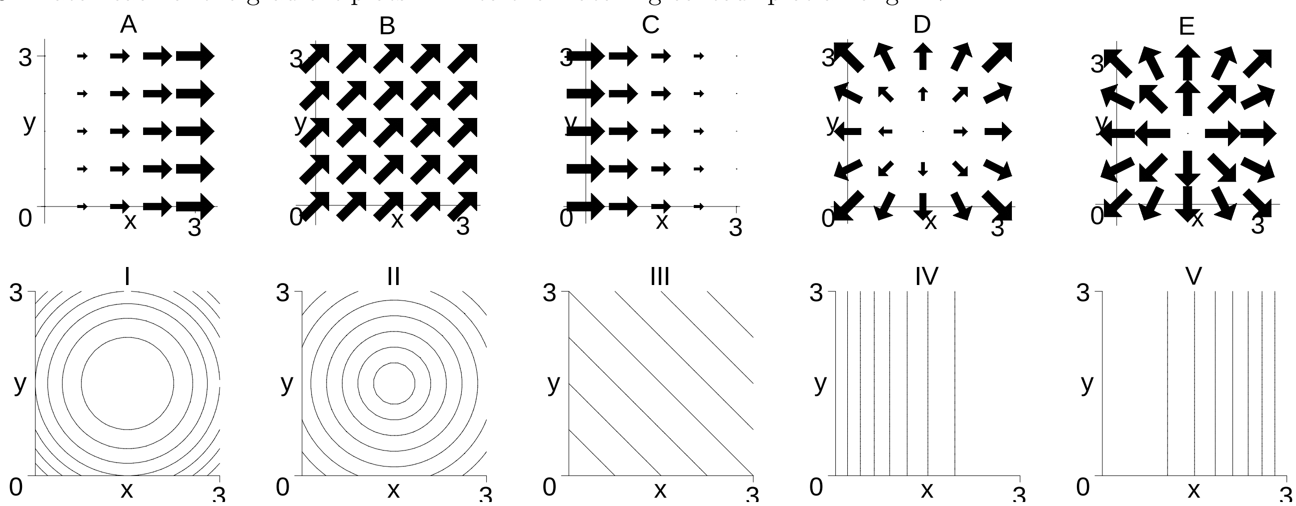
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$ when $z = \tan(x^2 - y^2), x = s/t$ and $y = \sqrt{t}$.

2. Fixing Maple errors. Each of the following produced an error or an empty graph, explain how to fix each. (Assume "with(plots);" has already been done.)

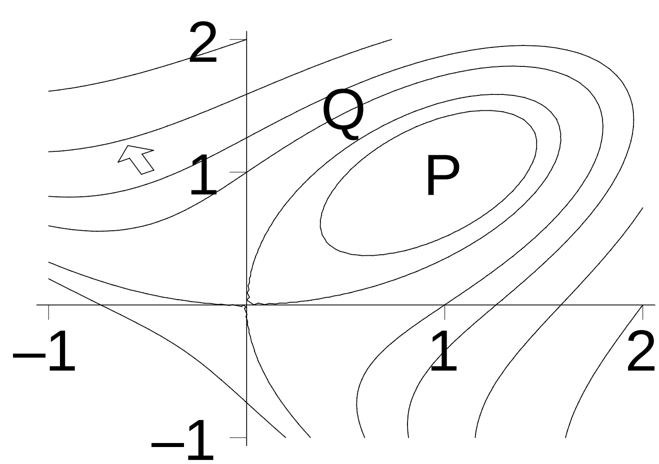
a `contourplot(sin(x*sin(y),x=0..Pi,y=0..Pi);`
b `gradplot(3y+2x,x=0..2,y=0..2);`
c `plot3d(exp(x+y),x=1..-1,y=-1..1);`
d `plot3d(x*y,x=0..1,y=0..10,scaling=contrained);`
e `contourplot(y^2+z^2,x=-10..10,y=-10..10);`

3. Match each of the gradient plots A–E to the matching contour plot among I–V.



4. Find the directional derivative of $f(x, y, z) = x^3 + y^2 + z$ as you leave the point $P(3, 2, 1)$ heading in the direction of the point $Q(0, 6, 1)$.

5. The graph below is a contour graph of a function $f(x, y)$ and the arrow at $(-\frac{1}{2}, 1)$ points in the direction of the gradient $\nabla f(-\frac{1}{2}, 1)$. Find and identify all local extrema (i.e. local mins, local maxs and saddles) and find the sign (positive, negative or zero) of the partials of f: $f_x(Q), f_y(Q), f_x(P), f_{xx}(P), f_x(0, 0)$ and the directional derivatives $f_{\mathbf{u}}(Q), f_{\mathbf{u}}(0, 0)$ for $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$



6. Find the equation of the tangent plane to the level surface $F(x, y, z) = x^2 + 2y^2 + 6xy - 8x^3z + 27 = 0$ at $(1, -2, 3)$.

7. Use your TI-89 to find all the critical points of the function $f(x, y) = x^3 - 3xy + y^3$, then show how you would obtain these critical points by hand. Classify these local extrema by filling out a table like the one below, with a separate line for each critical point.

(x, y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
?	?	?	?	?	?

8. Sketch the region and rewrite the cylinderical triple integeral below in both spherical and rectangular coordinates, do **NOT** evaluate. (Here $\delta = \delta(x, y, z) = \delta(z, r, \theta) = \delta(\rho, \phi, \theta)$ is some density function.)

$$\int_0^{\pi/2} \int_0^2 \int_{-\sqrt{4-r^2}}^0 \delta \; r \, dz \, dr \, d\theta$$

[Hint: Draw both the 2D area in the xy-plane that the 3D region "lives" over and a 2D z vs r plot.]

9. Sketch the region of integration of the polar coordinate integral

$$\int_0^{\pi/4} \int_0^2 r^2 \cos\theta \, dr \, d\theta$$

and rewrite it as rectangular integral (or a sum of integrals) in **BOTH** orders $dx\,dy$ and $dy\,dx$. Evaluate all three integrals (or sum of integrals) on your TI-89.

10. Use Lagrange multipliers to find the minimum and maximum VALUES of $2x^2 - y^2 + z$ subject to the constraint $x^2 + y^2 + z^2 = 1$. [Hint: there are six "critical points".]