MAC 2313 Calculus 3 **Test 3** 20 Nov 2003

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

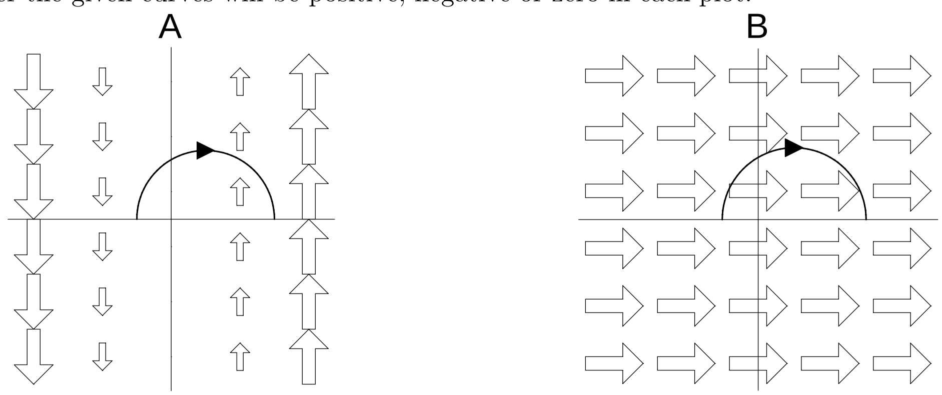
1. Find curl $\vec{F}$ and find div $\vec{G}$, if the vector field $\vec{F} = \left\langle x^2y^2z^2, \pi, xyz \right\rangle$ and if the vector field $\vec{G} = \left\langle \sin(x^2+y^2), (3y+z)^{100}, 5z \right\rangle$.

2. Vector, scalar or nonsense. $f = f(x,y,z)$ and $g = g(x,y,z)$ are scalar fields and $\vec{F} = \vec{F}(x,y,z)$ and $\vec{G} = \vec{G}(x,y,z)$ are vector fields. Determine if the given object is a scalar field, a vector field or nonsense.

A. $f+g$	B. $\vec{F}+\vec{G}$	C. $f+\vec{G}$	D. $\nabla\vec{F}$	E. div g
F. curl(curl $\vec{G}$)	G. curl(div $\vec{G}$)	H. div(curl $\vec{F}$)	I. grad(div $\vec{F}$)	J. div(grad g)

3. Find a function f so that $\nabla f = \vec{F}$ and use it to compute the line intergal $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F} = \left\langle y, x+6y+z^2, 2yz+1/z \right\rangle$ where C is some very complex counterclockwise corkscrew curve going from $(1,3,2)$ to $(5,-1,3)$.

4. Find formula's for the two vector fields below. (There are many possible answers.) Decide if the line integrals over the given curves will be positive, negative or zero in each plot.



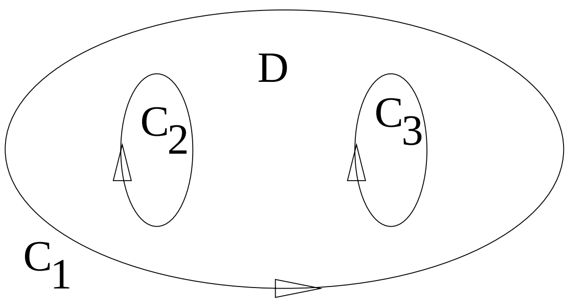
5. Compute the line intergal $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F} = \langle y, z, x \rangle$ and C is the is the line from $(1,3,2)$ to $(5,-1,3)$.

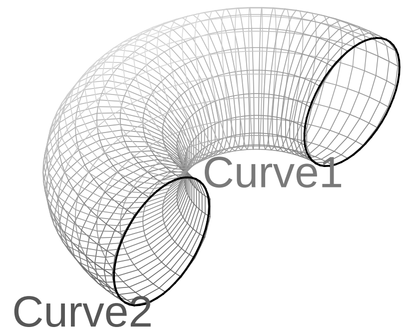
6. Compute the flux of the vector field $\vec{F} = y\hat{\imath} + (1+z)\hat{k}$ through the part of the plane $3x+3y+z=3$ oriented outward with $x \geq 0, y \geq 0, z \geq 0$

7. Use your TI-89 to graph the closed curve C given by $\vec{r}(t) = \left\langle \cos^3 t, \sin^3 t \right\rangle$ for $0 \leq t \leq 2\pi$. For the vector field $\vec{F} = \langle 0, x \rangle$ write and simplify the line integeral $\oint_C \vec{F} \cdot d\vec{r}$ down to a regular integral in t. Use your TI-89 to evaluate this integral. Finally use Green's theorem to explain why your answer is the area of the region enclosed by C.

There is more test on the other side.

8. The domain D of area 13 is the region inside C_1 but outside C_2 and C_3 (see below left) and $\vec{F} = \langle 7x^2 - 5y, 3x + 11y^2 \rangle$, find the line integral $\oint_{C_1} \vec{F} \cdot d\vec{r}$ if $\oint_{C_2} \vec{F} \cdot d\vec{r} = -15$ and $\oint_{C_3} \vec{F} \cdot d\vec{r} = -25$.





9. The outward oriented half-torus H (see above right), with parametric equation given by $\vec{r}(s,\theta) = \langle (2+\cos s)\cos\theta, (2+\cos s)\sin\theta, \sin s \rangle$, has been cut in half (from a full torus) along the yz-plane, and hence $0 \le s \le 2\pi$ and $\pi/2 \le \theta \le 3\pi/2$ is our range on the parameters. Let Curve1 and Curve2 be the circles in the picture whose union is ∂H and let $\vec{u} = \frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial \theta}$. The parameter θ is the same as in polar/cylindrical/spherical coordinates. Eventually, $\vec{u} = (2+\cos s)\langle -\cos s\cos\theta, -\cos s \sin\theta, -\sin s \rangle$.

AB. Show $||\vec{u}|| = 2 + \cos s$.

C. Which of $\vec{u}$ or $-\vec{u}$ points in the orientation of H?

D. If Curve1 is orientated as required by Stokes Theorem, is it clockwise or counterclockwise as we are looking at the picture?

E. If Curve2 is orientated as required by Stokes Theorem, is it the same or the opposite direction from the orientation of Curve1?

FG. Why is the following "application" of the diverence theorem false? Since $\vec{F} = \langle 1, 0, 0 \rangle$ is a vector field with zero diverence everywhere, then flux of $\vec{F}$ through H must be zero. (Note you are not asked to "compute" anything for this part.)

HIJ. Write an integral which will compute the surface area of H, and evaluate it.

10. Divergence Theorem. Let V be volume inside the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 3$. The boundry of V is oriented with outward pointing normal and divided into pieces as follows. Let B be the surface on the bottom of the cylinder, T be the surface at the top of the cylinder and S be the surface at the side of the cylinder. (See graphs below.) The vector field is $\mathbf{F} = \langle xy^2, xz^2, x^2 z \rangle$ compute the following three integrals, the first without integrating, the last two by converting to polar or cylindrical coordinates.

$$\iint_B \mathbf{F} \cdot d\mathbf{A} \qquad \iint_T \mathbf{F} \cdot d\mathbf{A} \qquad \iiint_V \text{div } \mathbf{F} \, dV$$

Using the Divergence Theorem you can use the information above to find the value of the flux over S. Do so, and also write the and simplify the flux integral over S. Do **NOT** evaluate this last integral. [Well, using the TI-89 to evaluate the integral would be a way of checking your answer.]