MAC 2313 Calculus 3 **Test 2** 22 Sep 2004

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Simplify all answers to this problem. A space curve has velocity vector given by $\vec{v}(t) = \langle t^2, t\sqrt{2}, 1\rangle$
   (a) find the position vector $\vec{r}(t)$ if $\vec{r}(1) = \langle 1, \sqrt{2}, 3\rangle$
   (b) find the acceleration $\vec{a}(t)$.
   (c) find the speed
   (d) find the (exact) arclength from $t = 1$ to $t = 5$
   (e) find a_N using $a_N = |\vec{r}\,'(t) \times \vec{r}\,''(t)|/|\vec{r}\,'(t)|$
   (f) find κ using $\kappa = |\vec{r}\,'(t) \times \vec{r}\,''(t)|/|\vec{r}\,'(t)|^3$
2. Write the equation $x^2+y^2 = z^2/25$ in cylindrical coordinates and write the equation $x^2+y^2+z^2+2z = 0$ in spherical coordinates. Simplify your answers.
3. A particle moves at a constant speed along a line frome the point $P = (1, -1, 2)$ to point $Q = (4, 1, -4)$. Find the parametric equations of the line and the limits $a \le t \le b$ if
   (a) It takes five seconds to go from P to Q.
   (b) The speed of the particle is 5 units per second.
4. Find the vectors $\vec{T}, \vec{N}$, and $\vec{B}$ for the curve $\vec{r}(t) = \langle t, \cos(3t), \sin(3t)\rangle$ at the point $(\pi/6, 0, 1)$. [Since we are at a point, the final vectors $\vec{T}, \vec{N}$, and $\vec{B}$ should be independent of t.]
5. Match the parametric equations in 1–6 to the graphs labeled A–F. Give reasons for your choices.

   (1) $\vec{r}(t) = \langle \cos t, \sin t, \sin 4t\rangle$ (2) $\vec{r}(t) = \langle t, \cos t, \sin t\rangle$ (3) $\vec{r}(t) = \langle \cos t, \sin t, t\rangle$

   (4) $\vec{r}(t) = \langle t\cos t, t\sin t, t\rangle$ (5) $\vec{r}(t) = \langle \sin t, t, \cos t\rangle$ (6) $\vec{r}(t) = \langle \cos t, -\sin t, t\rangle$