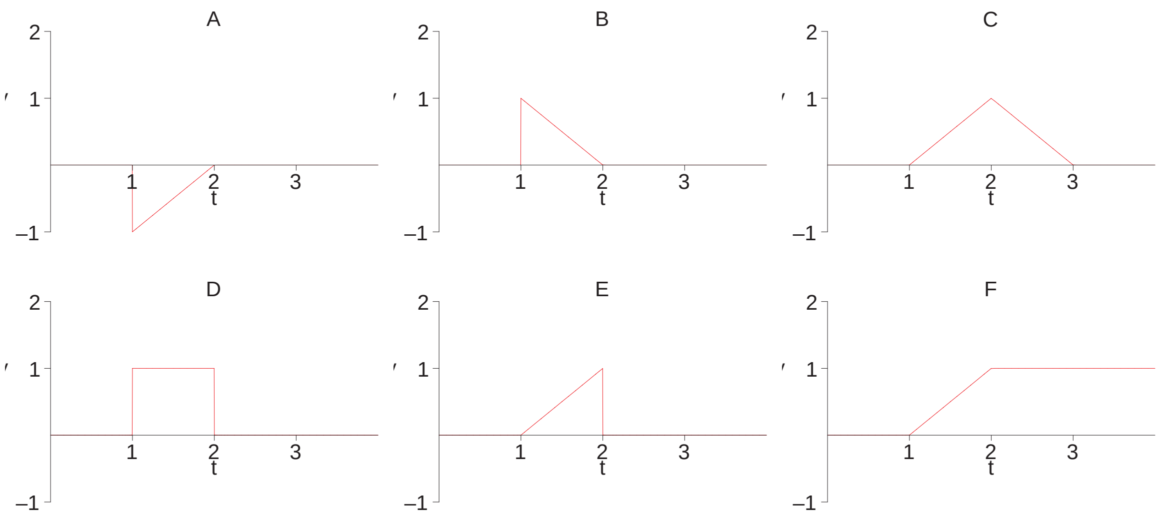
MAP 2302 Diff-E-Qs **Extra Laplace Problems** Summer 2005

1. Match the equations $y_1(t) = u_1(t) - u_2(t)$, $y_2(t) = (2-t)(u_1(t) - u_2(t))$, $y_3(t) = (t-1)u_1(t) - (t-2)u_2(t)$, $y_4(t) = (1 - |t|)(u_1(t) - u_3(t))$, $y_5(t) = (t-1)(u_1(t) - u_2(t))$, $y_6(t) = (2-t)u_1(t) + (t-3)u_2(t)$, and $y_7(t) = (t-2)u_1 - (t-2)u_2(t)$ to the graphs $A - F$ below and draw the missing graph.



2. True or False and a brief reason why or why not.

   (a) If $c = \max\{a, b\}$, then $u_a(t)u_b(t) = u_c(t)$

   (b) If $c = \max\{a, b\}$, then $\delta(t-a)\delta(t-b) = \delta(t-c)$

   (c) For constants a and b and for $t \geq 0$ the solutions to the IVP $y' + ay = 0, y(0) = b$ and the IVP $y' + ay = b\delta(t), y(0) = 0$ have the same solution.

   (d) For constants a, b and c, and for $t \geq 0$ the solutions to the IVP $y'' + ay' + by = 0, y(0) = c, y'(0) = 1$ and the IVP $y'' + ay' + by = \delta(t), y(0) = c, y'(0) = 0$ have the same solution.

   (e) When $0 < a < b$, and $\mathcal{L}[h(t)] = H(s)$ then the inverse Laplace transform of $(\exp(-as) - \exp(-bs))H(s)$ is $(u_a(t) - u_b(t))h(t-a)$.

   (f) $\mathcal{L}^{-1}[(s+2)/((s+1)^2 + 4)] = e^{-t}\cos 2t$

   (g) $\int_{-\infty}^{\infty} \delta(t)\sin(2t)/t\, dt = 2$

   (h) $\int_{-\infty}^{\infty} u_a(t) - u_b(t)\, dt = b - a$

   (i) $\lim_{t \to c^-} u_c(t) = 1$

   (j) $|t| = tu_0(t) - t$