MAC 3313 Calculus 3 **Test 2** 22 Oct 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. For $f(x, y) = x^3 - 4x^2 + y^2$ and $\mathbf{u} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$, find ∇f, and $D_{\mathbf{u}} f$.

2. Use the chain rule to find $\frac{\partial w}{\partial t}, \frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, if $w = x^2 + y^2 + z^2, x = t \sin v \cos u, y = t \sin v \sin u$, and $z = t \cos u$

3. Find the equation of the tangent plane to $f(x, y) = x^2y + xy^2$ at $(2, 4)$.

4. Show the limit $\lim_{(x,y,z)\to(0,0,0)} \frac{x^2-y^2-z^2}{x^2+y^2+z^2}$ does not exist.

5. Set up but do **NOT** evaluate the interated integral (or sum of interated integrals) which will give the volume under the paraboloid $z = 3x^2 + y^2$ and above the region bounded by $y = x$ and $x = y^2 - y$.

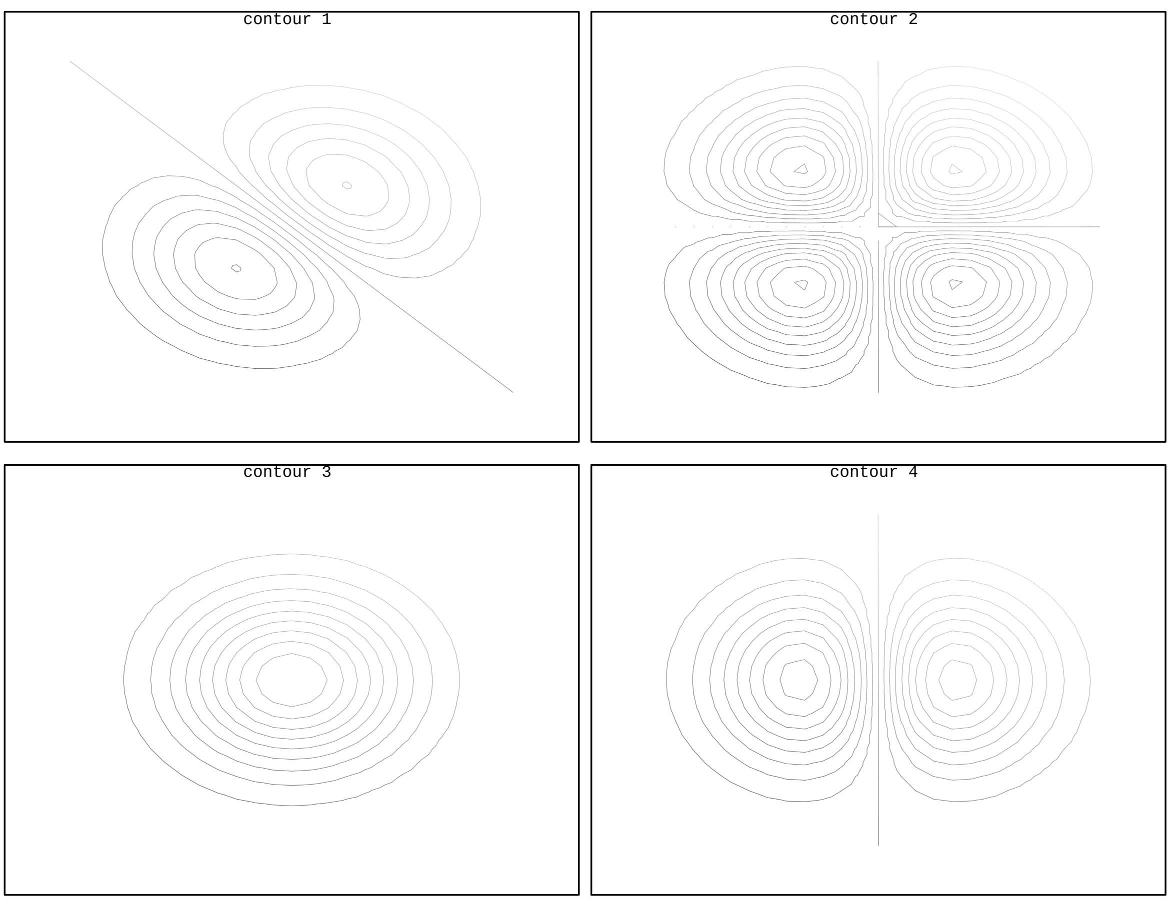
6. The function $f(x, y) = x \sin y$ has infinitely many critical points. Find them and determine if they are local minimums, local maximums or saddle points.

7. Sketch the region of integration and change the order of integration of $\int_0^1 \int_{y^2}^{2-y} f(x, y)\, dxdy$.

8. Compute the mass of the lamina of the triangular region with vertices $(0, 0), (1, 1)$ and $(4, 0)$ with density $\rho(x, y) = x$. (Integrating with respect to x first produces less integrals.)

9. Find the points on the lines $\langle 1, 1, t\rangle$ and $\langle 3 + s, 0, -s\rangle$ closest to each other.

10. Let $b(x, y) = \exp(-x^2 - y^2)$. Below are Maple contour plots of the functions (in some order) of $b(x, y), xb(x, y), xyb(x, y)$ and $(x + y)b(x, y)$ over the range x=-2..2, y=-2..2. Identify which is which.



Maple contour plots