MAC 2313 Calculus 3 **Test 3** 24 Nov 1998

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only.

1. Find the curl and div of $\mathbf{F} = \langle zx, xy, yz \rangle$.

2. Find f so that $\mathbf{F} = \nabla f$ and use it to find the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$. Here $\mathbf{F} = \left\langle 4xe^z, \cos y, 2x^2 e^z \right\rangle$ and C is a curve from $(0,0,0)$ to $(1,\pi,2)$.

3. Write down and simplify but do **NOT** evaluate the double integral obtained from using Green's Theorem to change $\int_C (y + e^{\sqrt{x}})dx + (3x^2 + \cos y^2)dy$ into a double integral. C is the boundry of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

4. Write down and simplify but do **NOT** evaluate the quotient of of the two integrals needed to find the z coordinate, $\bar{z}$, of the center of mass of the volume between $z = y$ and $z = 2 + x$ over the unit square $0 \le x, y \le 1$ if the density is given by $\rho(x,y,z)$.

5. Let f be a scalar field and $\mathbf{F}$ a vector field. State whether each expression is a scalar field, a vector field or meaningless. A. curl f B. grad f C. div $\mathbf{F}$ D. curl(grad f) E. grad $\mathbf{F}$ F. grad(div $\mathbf{F}$) G. div(grad f) H. grad(div f) I. curl(curl $\mathbf{F}$) J. div(div $\mathbf{F}$)

6. Write down and simplify but do **NOT** evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$. Where S is the hemisphere $z = \sqrt{16 - x^2 - y^2}$ with upward normal and $\mathbf{F} = \langle -y, x, 3z \rangle$.

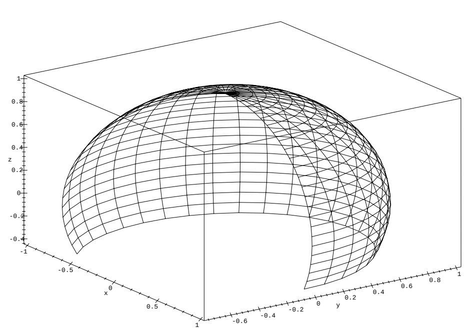
7. Evaluate and **SIMPLIFY** $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} 70xz \quad dzdxdy$ by changing to cylindrical coordinates.

8. Sketch the region of integration and re-write the integral $\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z) \quad dzdydx$ in the order $dxdydz$.

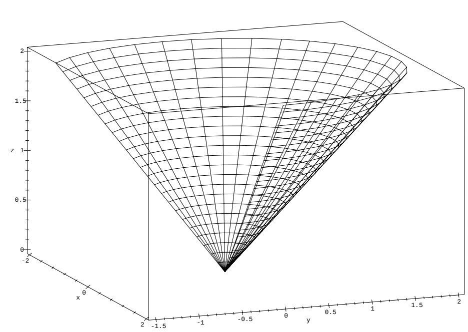
9. Use the transformation $x = u^2, y = v^2$ to find the area of the region R bounded by the curve $\sqrt{x} + \sqrt{y} = 1$ and coordinate axes. Explicitly draw R and S, the region in the uv plane that maps to R in the xy plane by this transformation. Clearly label the Jacobian of the transformation.

10. Match the parameteric equations $\mathbf{r}(u,v) = \langle \cos v, \sin v, u \rangle$, $\mathbf{r}(u,v) = \langle (2+u)\cos 2v, (2+u)\sin 2v, v \rangle$, $\mathbf{r}(u,v) = \langle u\cos v, u \sin v, u \rangle$, and $\mathbf{r}(u,v) = \langle \sin u \cos v, \sin u \sin v, \cos u \rangle$ with the Maple plot3d's below. The range is always $u = 0..2, v = 0..4$.

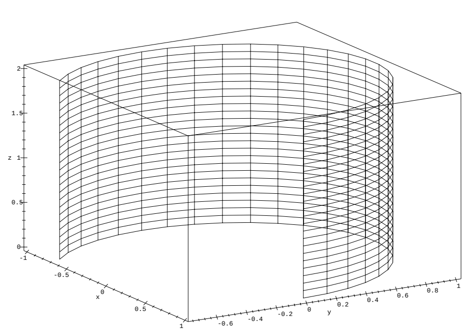
surface1



surface2



surface3



surface4