MAC 2313 Calculus 3 **Test 2** 2 Mar 2000

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find the equation of the tangent plane to $z = \sqrt{17 - x^2 - y^2}$ at $(x, y) = (3, 2)$

2. Find a vector parallel to the intersection of the planes $2x - 3y + 5z = 2$ and $4x + y - 3z = 7$.

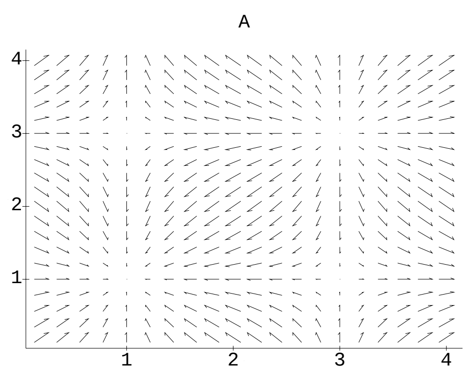
3. Find the directional derivative of $z = x^2 - y^2$ at the point $(3, -1)$ in the direction making an angle $\theta = \pi/4$ with the x-axis. In which direction is the directional derivative the largest?

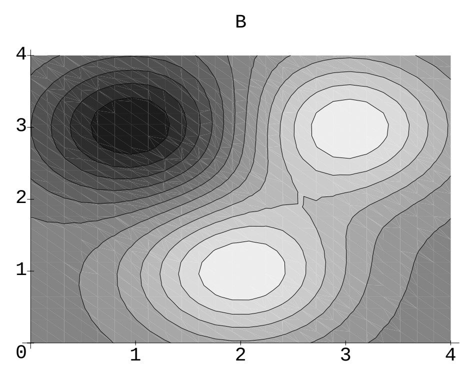
4. A table of the critical points of the function $f(x, y)$ is given below along with values of various partial derivatives at these points. Also these values for the non-critical point $(0, 0)$ are given. Give the quadratic Taylor polynomial for the function f about $(0, 0)$. Also for each critical point of f decide if the point is a local mimimum, local maximum or a saddle point.

(x, y)	$f(x, y)$	$f_x(x, y)$	$f_y(x, y)$	$f_{xx}(x, y)$	$f_{xy}(x, y)$	$f_{yy}(x, y)$
$(0, 1)$	0	0	0	1	2	0
$(0, -1)$	0	0	0	0	-2	0
$(1, 0)$	-4	0	0	2	0	2
$(-1, 0)$	4	0	0	-2	0	-2
$(0, 0)$	2	-1	1	2	4	-6

5. Use the Chain Rule to find $\partial w/\partial u, \partial w/\partial u$ at $(u, v) = (1, \pi)$ and dw/dt at $t = 1$, if $w = f(x, y, z) = 3xy + yz, x = \ln u + \cos v, y = 1 + u \sin v, z = uv, u = 1 + \sin(\pi t)$ and $v = \pi t^2$.

6. The graph A is a plot of ∇f, the gradient of f and the graph B is a contourplot of g. (Light regions have higher values than dark regions.] Find the co-ordinates of all extrema of f and g and **LABEL** them as either local minimums, local maximums or saddle points.





7. Use Lagrange Multipliers to find the maximum and minimum **VALUES** of $-3x^2 - 2y^2 + 20xy$ on the line $x + y = 100$, if they exist. If one or both don't exist explain why.

8. A table of the function $z = f(x, y)$ is below.

	$x = 3$	$x = 4$	$x = 5$	$x = 6$	$x = 7$
$y = 0$	0.6	0.0	0.6	1.3	3.0
$y = 1$	1.0	0.8	1.0	2.0	3.8
$y = 2$	3.0	2.6	3.0	4.0	5.6

AB. Find the Riemann sums which are the reasonable over- and under-estimates for the double integral $\int_R f(x, y)dA$ where the rectangle $R = \{(x, y) : 3 \le x \le 7, 0 \le y \le 2\}$.

C. Find the best estimate of the directional derivative of f at $(5, 1)$ in the direction $(\mathbf{i} + \mathbf{j})/\sqrt{2}$.

D. Find the best estimate of the directional derivative of f at $(5, 1)$ in the direction $(-2\mathbf{i} + \mathbf{j})/\sqrt{5}$.

9. Maple questions.

A. What Maple command, or sequence of commands would you use to find the expression below?

$$\frac{\partial^3}{\partial x \partial y^2} x \sin(y)$$

B. What Maple command, or sequence of commands would you use to find the solution(s) to the system of equations below?

$$x + y = 5$$
$$x - y = 1$$

C. What Maple command, or sequence of commands would you use to find the double integral below?

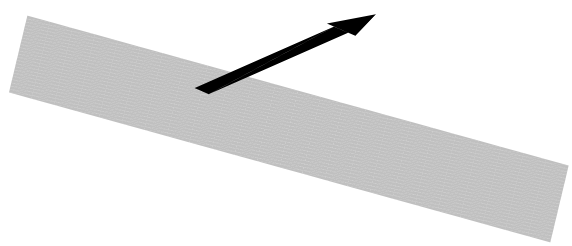
$$\int_0^1 \int_{x^2}^{x} x^{100} y^2 \mathrm{d}y \mathrm{d}x$$

D. What is wrong and how do you fix the Maple command below?

```
plot(x*exp^x,x=-4..1,title=‘surprise‘,color=green);
```

E. You execute the Maple commands below. The picture (like below) shows the "normal vector" on top of a plane but the normal vector doesn't look perpendicular to the plane. How do you fix this?

```
with(plots):with(plottools):
a:=plot3d(3-x-y,x=0..3,y=0..3):b:=arrow([1,1,1],[2,2,2],.2,.4,.2):
display(a,b);
```



10. A circular city has radius r km and an average population density of ρ people/km^2. In 1997 the population was 3 million, the radius was 25 km and growing at 0.1 km/year. If the density was increasing at 200 people/km^2/year, find the rate at which the total population was growing.