MAC 2312 Calculus 2 **Test 3** 12 Apr 2001

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

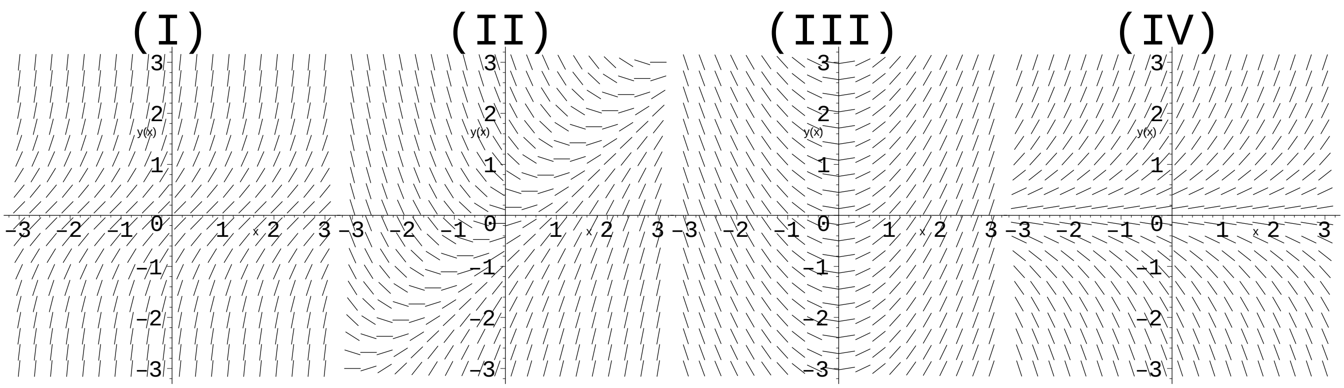
1. Solve the IVP

$$\frac{dy}{dx} + xy^2 = 0, y(1) = 1$$

2. Find the general solution to the differential equation $y'' + 6y' + 8y = 0$.

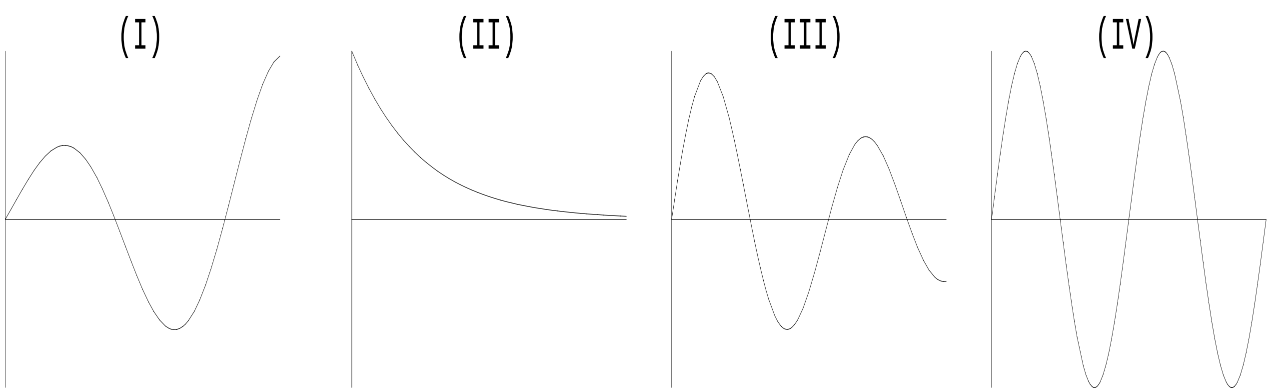
3. Find all values of $r$ so that $y = e^{rt}$ is a solution to $y''' - 9y' = 0$?

4. Match the slope fields below with the differential equations $y' = 1 + y^2, y' = y, y' = x$ and $y' = x - y$.




5. Find the range of values on $b$ which will make the general solution to the ODE $s'' + bs' + 5s$, overdamped? underdamped? and critically damped?

6. Match the graphs of the solutions below with the differential equations $y'' + 4y = 0, y'' - 4y = 0, y'' - 0.2y' + 1.01y = 0$ and $y'' + 0.2y' + 1.01y = 0$.




7. In the project you used a geometric series to estimate the error of truncating a power series to only $N$ terms. Repeat this process to find an estimate of the error of the special case of using

$$\sum_{n=0}^{10} \frac{5^n}{n!}$$

instead of the exact $e^5 = \sum_{n=0}^{\infty} 5^n/n!$. Be sure to explicitly give $a$ and $r$ and how you got them.

8. Consider the IVP

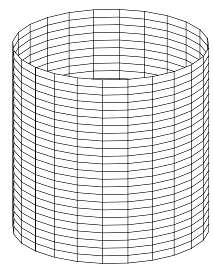
$$y' = 5 - y, y(0) = 1$$

(a) Use Euler's method by hand with two steps to estimate $y(1)$.
(b) Sketch the slope field for this ODE in the first quadrant, and use it to decide if your estimate is an over- or underestimate.
(c) Use Euler's method via your calculator to estimate $y(1)$ with ten steps.
(d) Use Euler's method via your calculator to estimate $y(1)$ with twenty steps.

9. A cup of java (below left) is made with boiling water and stands in a room where the temperature is $20°C$.
(a) If $T(t)$ is the temperature of the coffee at time $t$, explain what the ODE

$$\frac{dT}{dt} = -k(T - 20)$$

says in everyday terms. What is the sign of $k$?
(b) Solve this ODE. If the coffee cools to $90°C$ in 2 minutes, how long will it take to cool to $60°C$?



10. Derive but do **NOT** solve the differential equations below.
A $30m$ tall upright cylinderical tank (above right) has a circular base with area $100m^2$ and initially it has $1000m^3$ of fresh water (so initially the height of the water is $10m$). Into this large tank water flows a $3m^3/minute$ salt water solution containing 20 kilograms of salt per $m^3$. The solution is kept uniform in the tank by stirring. and the mixed water flows out according to the rules given below.

Case I: Assume the mixture is also leaving the tank at the same $3m^3/minute$ rate so that the $1000m^3$ volume remains constant.
(a) Derive a differential equation and initial values for $Q$ the quantity of the salt in the tank (in $kg$).

Case II: Assume the water mixture now flows out at a rate proportional to the square root of the height $h(t)$ of the water instead of the constant rate in Case I.
(b) Derive a differential equation and initial values for $h$ the height of the water in the tank. (Remember there is water flowing in as well as water flowing out.)
(c) Derive a differential equation and initial values for $Q$ the quantity of the salt in the tank (in $kg$) using this second outflow assumption. You can use the $h = h(t)$ from part (b) in your equation.