Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Solve the IVP
   \[
   \frac{dy}{dx} + xy^2 = 0, \quad y(1) = 1
   \]

2. Find the general solution to the differential equation \( y'' + 6y' + 8y = 0 \).

3. Find all values of \( r \) so that \( y = e^{rt} \) is a solution to \( y'' - 9y' = 0 \).

4. Match the slope fields below with the differential equations \( y' = 1 + y^2, \ y' = y, \ y' = x \) and \( y' = x - y \).

5. Find the range of values on \( b \) which will make the general solution to the ODE \( s'' + bs' + 5s \), overdamped? underdamped? and critically damped?

6. Match the graphs of the solutions below with the differential equations \( y'' + 4y = 0, \ y'' - 4y = 0, \ y'' - 0.2y' + 1.01y = 0 \) and \( y'' + 0.2y' + 1.01y = 0 \).

7. In the project you used a geometric series to estimate the error of truncating a power series to only \( N \) terms. Repeat this process to find an estimate of the error of the special case of using
   \[
   \sum_{n=0}^{10} \frac{5^n}{n!}
   \]
   instead of the exact \( e^5 = \sum_{n=0}^{\infty} \frac{5^n}{n!} \). Be sure to explicitly give \( a \) and \( r \) and how you got them.
8. Consider the IVP
\[ y' = 5 - y, \ y(0) = 1 \]
(a) Use Euler’s method by hand with two steps to estimate \( y(1) \).
(b) Sketch the slope field for this ODE in the first quadrant, and use it to decide if your estimate is an over- or underestimate.
(c) Use Euler’s method via your calculator to estimate \( y(1) \) with ten steps.
(d) Use Euler’s method via your calculator to estimate \( y(1) \) with twenty steps.

9. A cup of java (below left) is made with boiling water and stands in a room where the temperature is 20°C.
(a) If \( T(t) \) is the temperature of the coffee at time \( t \), explain what the ODE
\[ \frac{dT}{dt} = -k(T - 20) \]
says in everyday terms. What is the sign of \( k \)?
(b) Solve this ODE. If the coffee cools to 90°C in 2 minutes, how long will it take to cool to 60°C?

10. Derive but do NOT solve the differential equations below.
A 30m tall upright cylindrical tank (above right) has a circular base with area 100m² and initially it has 1000m³ of fresh water (so initially the height of the water is 10m). Into this large tank water flows a 3m³/minute salt water solution containing 20 kilograms of salt per m³. The solution is kept uniform in the tank by stirring, and the mixed water flows out according to the rules given below.
Case I: Assume the mixture is also leaving the tank at the same 3m³/minute rate so that the 1000m³ volume remains constant.
(a) Derive a differential equation and initial values for \( Q \) the quantity of the salt in the tank (in kg).
Case II: Assume the water mixture now flows out at a rate proportional to the square root of the height \( h(t) \) of the water instead of the constant rate in Case I.
(b) Derive a differential equation and initial values for \( h \) the height of the water in the tank. (Remember there is water flowing in as well as water flowing out.)
(c) Derive a differential equation and initial values for \( Q \) the quantity of the salt in the tank (in kg) using this second outflow assumption. You can use the \( h = h(t) \) from part (b) in your equation.