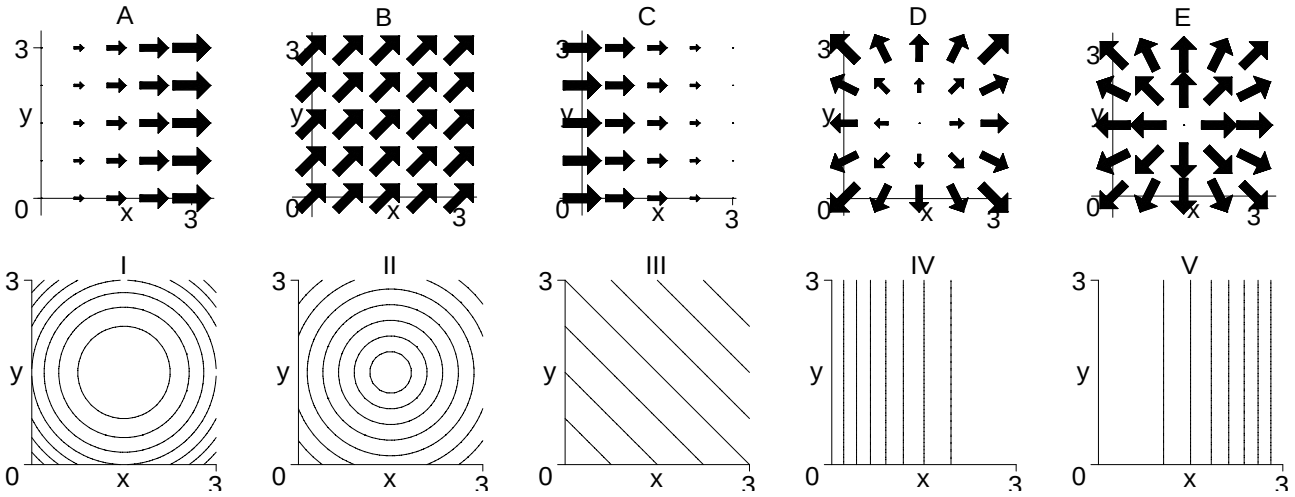
MAC 2313 Calculus 3 **Test 2** 5 Mar 2002

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find the equation of the tangent plane to $x + y^2 + z^3 + xyz = 14$ at $(x, y, z) = (3, 2, 1)$

2. Find the direction derivative, $F_{\mathbf{u}}(\pi/4, \pi/3, 3)$, if $F(x, y, z) = \sin x \cos y \ln z$ and $\mathbf{u}$ is the unit vector in the direction of $\langle 4, -4, 2\rangle$.

3. Match each of the gradient plots A–E to the matching contour plot among I–V.

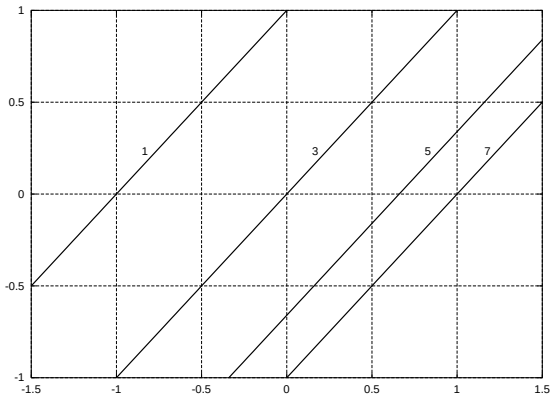


4. Use the Chain Rule to find $\partial z/\partial u$ and $\partial z/\partial v$ when $z = (x + y)\ln(x + y), x = u \sin v$ and $y = u^2 + v^{-2}$.

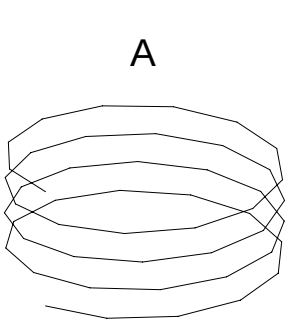
5. Find the local extrema of the function $f(x, y) = 8xy - \frac{1}{4}(x + y)^4$. [Hint: Use your TI-89 to check that you got the correct collection of critical points.]

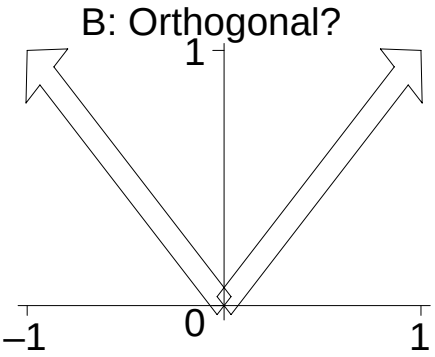
6. Let $\mathbf{r}(t) = \langle t\cos(2\pi t), t\sin(2\pi t)\rangle$.
(a) Find the velocity and acceleration of $\mathbf{r}(t)$.
(b) Plot the curve for $0 \le t \le 2$ [Hint: your TI-89 can do this].
(c) Write down an integral which will give the arclength of the curve in (b). Use the TI-89 to find a symbolic answer to the integral. (You may have to simplify before integrating.)
(d) Find a numerical approximation to the integral in (c).

7. The function f has the contour plot given below. Give your best estimates of $f_x(0, 0)$ and $f_y(0, 0)$ (carefully showing the data points you used) and determine the sign (positive, negative or zero) of the second derivatives $f_{xx}(0, 0), f_{yy}(0, 0)$, and $f_{xy}(0, 0)$.



8. Use Lagrange Multipliers to find the maximum and minimum **VALUES** for $f(x, y) = x + 2y$ on the ellipse $x^2 + 4y^2 = 1$.



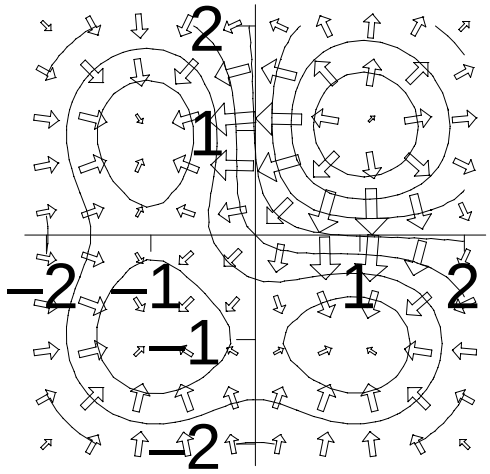


9. Extreme Maple.

(a) What are the problems with graph A and graph B (above) and how to you fix them (in Maple)?

(b) What sequence of Maple commands would you use to make graph similar to the one below if f is the expression to be plotted. (The defaults give "vectors" or "arrows" that look different and different size fonts and that is ok. You can assume "with(plots);" is already done.)

(c) List the locations of all the local minimums, local maximums and saddles in the graph below.



10. This problem is to find the critical points of the function $z = F(x, y) = f(x) + g(y)$ and classify them. Make a table and for each critcal point, give the sign (positive, negative or zero) of each of the second partials F_{xx}, F_{xy} and F_{yy} the sign of the discriminant D and the classification. (If more than one classification is possible, then give all possible classifications.)

(x, y)	F_{xx}	F_{yy}	F_{xy}	D	Classification
?	?	?	?	?	?

The function $z = f(x)$ has two critical points at $x = 0$ and $x = 1$ and the second derivative test for one variable says $x = 0$ is a local maximum and $x = 1$ is a local minimum. The function $z = g(y)$ has three critical points and the second derivative test for one variable says $y = 3$ and $y = 5$ are local minimums and $y = 4$ is a local maximum.