MAC 2313 Calculus 3 **Test 3** 11 Apr 2002

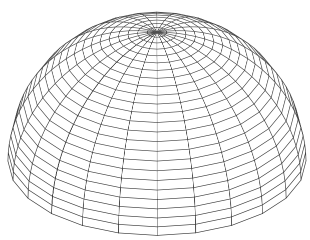
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

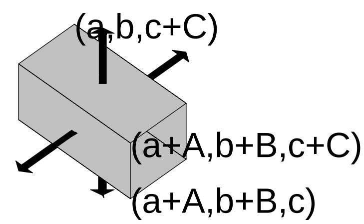
1. Find curl **F** and find div **G**, if the vector field $\mathbf{F} = \langle z - 2y, x - 3z, y - 4x \rangle$ and if the vector field $\mathbf{G} = \left\langle e^{xy}, z \ln y, xz^2 - zy^2 \right\rangle$.

2. Find a scalar field f so that $\nabla f = \mathbf{F} = \langle 2x + y + z, x + 2y, x + 2z \rangle$ and use it to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{r}(t)$ is an extremely complex helical spiral that goes from $(1, 0, 3)$ to $(2, -1, 5)$

3. Let C be the curve $\mathbf{r}(t) = \left\langle t^2, 2 - t \right\rangle$ for $0 \le t \le 1$ and let $\mathbf{F} = \left\langle -y^2, x \right\rangle$. Compute the line integral, $\int_C \mathbf{F} \cdot d\mathbf{r}$.

4. Write down triple integrals in each of the cordinate systems spherical, cylinderical and rectangluar will compute the mass of the hemispherical region (below left) given by $x^2 + y^2 + z^2 \le R^2$ and $z \ge 0$ with density function δ. Do **NOT** evaluate. [You can't since δ isn't defined.]





5. Compute the flux of the vector field $\mathbf{F} = \langle x, 0, 0 \rangle$ through each of the rectangular regions S in (a)–(d).
   a. S is a horizontal rectangle with one corner at $(a, b, c+C)$ and the opposite corner at $(a+A, b+B, c+C)$ oriented in the positive z-direction. [Above right top rectangle]
   b. S is a horizontal rectangle with one corner at (a, b, c) and the opposite corner at $(a+A, b+B, c)$ oriented in the negative z-direction. [Above right bottom rectangle]
   c. S is a rectangle parallel to the yz-plane with one corner at $(a + A, b, c)$ and the opposite corner at $(a + A, b + B, c + C)$ oriented in the positive x-direction. [Above right front rectangle]
   d. S is a rectangle parallel to the yz-plane with one corner at (a, b, c) and the opposite corner at $(a, b + B, c + C)$ oriented in the negative x-direction. [Above right rear rectangle]

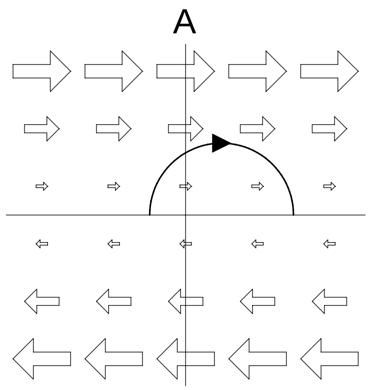
Let V be the rectangular solid given by $a \le x \le a + A$, $b \le y \le b + B$, $c \le z \le c + C$; so V is a box with faces parallel to the usual coordinate planes. Let S be the boundrary of V which is the six faces top, bottom, front, rear, left and right all orientated outward. You have computed the flux across 4 of the faces in (a)–(d) and the other two are 0. The divergence theorem states for a volume V and $S = \partial V$ with outward orientation:

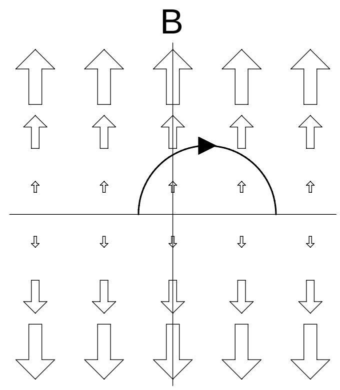
$$\iint_S \mathbf{F} \cdot d\mathbf{A} = \iiint_V \text{div } \mathbf{F} \; dV$$

The flux integral on the left hand side is just the sum of your answers to (a)–(d).
   e. Compute the right hand side of the above equation and hence show the Diverengce theorem is true for this case.

6. Sketch the region of integration and change the order of integration of $\int_1^2 \int_0^{\ln x} f(x, y) \; dy \; dx$

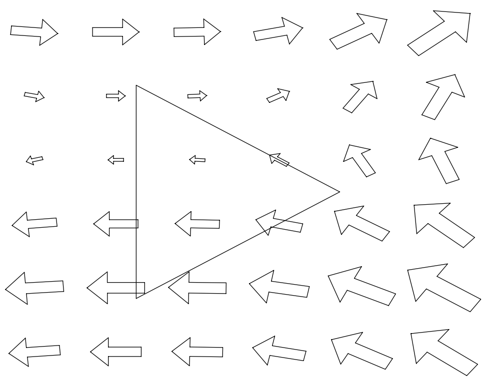
7. Find formula's for the two vector fields below. (There are many possible answers). Decide if the line integrals over the semi-circular curve will be positive, negative or zero for each vector field.

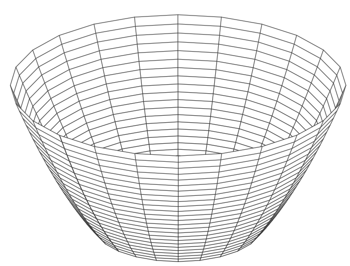




8. By computing both sides show div $(f\mathbf{F}) = \text{grad } f \cdot \mathbf{F} + f \text{ div } \mathbf{F}$. Here $f = f(x, y, z)$ is a scalar function and $\mathbf{F}$ is the vector field $\langle P(x, y, z), Q(x, y, z), R(x, y, z)\rangle$. [Hint $f\mathbf{F} = \langle fP, fQ, fR\rangle$.]

9. Use Green's theorem to convert the closed line integral $\oint_C \left\langle -\cos y, \sin^3 x\right\rangle \cdot d\mathbf{r}$ into a regular double integral if C is the counter-clockwise boundrary of the triangle with vertices $(0, 0), (0, 2)$ and $(1, 1)$. Use your TI-89 to evaluate the integral. (Picture below left)





10. Compute the flux integral below by converting the resulting regular double integral into polar coordinates.

$$\iint_S \mathbf{G} \cdot d\mathbf{A}$$

Where $\mathbf{G} = \langle xz, yz, 1\rangle$ and our surface (above right) S is $z = x^2 + y^2$ over the annulus $1 \leq x^2 + y^2 \leq 4$ oriented upward.