MAC 2313 Calculus 3 **Test 2** 27 Feb 2003

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Show the limit below does not exit.

$$\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$$

2. Use the Chain Rule to find $\partial z/\partial u$ and $\partial z/\partial v$ when $z = \ln(x^2+xy+y^2)$, $x = v/u$ and $y = u\sqrt{v}$.

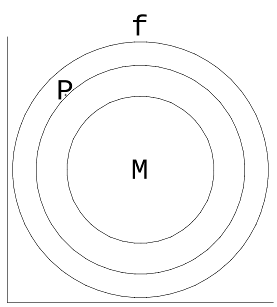
3. Fixing Maple errors. Each of the following produced an error or an empty graph, explain how to fix each. (Assume "with(plots);" has already been done.)

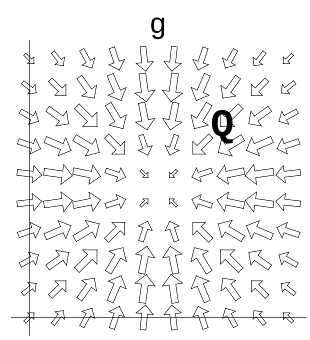
a `gradplot(x^2+xy+y^2,x=0..2,y=0..2);`
b `plot3d(exp(x+y),x:=0..1,y=-1..1);`
c `contourplot(x*y,x=-10..10,y=-10..10,contours=1, 4, 9, 25, 36]);`
d `plot3d(x*y,x=-1..1,y=-1..1,axes=boxed,title=my graph);`
e `contourploot(sin(x)*sin(y),x=0..Pi,y=0..Pi);`

4. Find the directional derivative of $f(x,y,z) = x^3y^2z$ as you leave the point $P(-1,2,1)$ heading in the direction of the point $Q(1,0,2)$.

5. The point P is on the contour graph of the function f with a local maximum at M (below left) and the point Q is on the gradplot of the graph of the function g (below right). Let $\mathbf{u}$ be the unit vector $\mathbf{u} = (\mathbf{i}+\mathbf{j})/\sqrt{2}$ and let $\mathbf{v}$ be the unit vector $\mathbf{v} = (\mathbf{i}-\mathbf{j})/\sqrt{2}$.

Find the sign (positive, negative or zero) of the partials of f: $f_x(P), f_y(P), f_{xx}(P), f_{yy}(P)$ and the partials of g: $g_x(Q), g_y(Q)$ and the four directional derivatives $f_{\mathbf{u}}(P)$, $f_{\mathbf{v}}(P)$, $g_{\mathbf{u}}(Q)$ and $g_{\mathbf{v}}(Q)$.



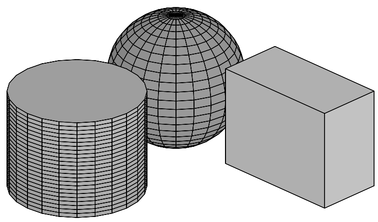


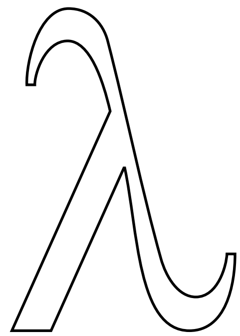
6. A differentiable function $f(x,y)$ has the property that $f(1,2) = 7$ and $\nabla f(1,2) = 2\mathbf{i} - 5\mathbf{j}$.

a. Find the equation of the tangent PLANE to the SURFACE $z = f(x,y)$ at the point $(1,2,7)$.
b. Find the equation of the tangent LINE to the level CURVE of f through the point $(1,2)$.

There is more test on the back.

7. Change the cylinderical coordinate equation $z = r$ into rectangular and spherical coordinates (simplify). Sketch the graph and describe the graph in words.





8. Use Lagrange multipliers to find the minimum and maximum VALUES of $x^2 - 2x + y^2 - 2y$ subject to the constraint $x^2 + y^2 = 2$.

9. Find critical points of the function $f(x, y) = x^3 + 3xy^2 - 12x$. Classify these local extrema by filling out a table like the one below, with a separate line for each critical point. [Hint: Use your TI-89 to check that you got the correct collection of critical points.]

(x, y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
?	?	?	?	?	?

10. Find the coordinates of the **POINT** closest to the point $P(1, 1, 0)$ which is both on the surface $z^2 = x^2 + y^2$ AND is in the first octant.