MAC 2313 Calculus 3 **Test 3** 17 Apr 2003

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find curl $\mathbf{F}$ and find div $\mathbf{G}$, if the vector field $\mathbf{F} = \langle 3x+2y, 3x+z, 2y+z\rangle$ and if the vector field $\mathbf{G} = \left\langle \ln(x/y), \ln(x/y), e^{-2z}\right\rangle$.

2. Vector, scalar or nonsense. $f = f(x,y,z)$ and $g = g(x,y,z)$ are scalar fields and $\mathbf{F} = \mathbf{F}(x,y,z)$ and $\mathbf{G} = \mathbf{G}(x,y,z)$ are vector fields. Determine if the given object is a scalar field, a vector field or nonsense.
A. div $\mathbf{F}$ B. div f C. curl $\mathbf{F}$ D. curl f E. $\mathbf{F}\cdot f$ (the dot product) F. (grad f) $\cdot$ $\mathbf{G}$
G. (curl$\mathbf{F}$) $\times$ $\mathbf{G}$ H. fg I. $\partial f/\partial x$ J. $f\cdot g$ (the dot product).

3. Write down a triple integral which will give volume of the the region under the surface $z = 1 + x + y^2$ above the triangle $T = \{(x,y) : 0 \le y \le 1, 0 \le x \le y\}$ in the xy-plane. Do **NOT** evaluate.

4. Sketch the region of integration and reverse the order of integration for

$$\int_0^2 \int_{x^2}^{2x} dy\ dx$$

[You do **NOT** have to evaluate the integrals, but using the TI-89 to evaluate both integrals would be a way of checking your answer.]

5. The vector field $\mathbf{F} = \langle 5,2,3\rangle$ compute the flux of $\mathbf{F}$ over each of the surfaces S described below without evaluation any integrals. When there is a choice of normal, pick the normal whose dot product with $\mathbf{F}$ is positive.
A. S is the rectangle $\{(0,y,z), 1 \le y \le 3, 0.5 \le z \le 1\}$.
B. S is a circle in the xz-plane with radius 5.
C. S is a pentagon of area P with normal in the direction of $\langle 5,-5,-5\rangle$

6. Use a theorem to show that if V is a 3D region in space with outward oriented boundry given by S then

$$\iint_S \langle x,y,z\rangle \cdot d\mathbf{A} = 3\ \text{Volume}\,(V)$$

7. Read this problem carefully. You are to sketch both of the vector fields and both the curves and explain the answers below but **NOT** by computing the line integrals. The two vector fields, let $\mathbf{F} = \langle x, 0\rangle$ and let $\mathbf{G} = \langle 0, x\rangle$. The two curves, let C_1 be the straight line from $(1,1)$ to $(3,1)$ and let C_2 be the circle or radius $2/\sqrt{\pi}$ centered at $(1,1)$ oriented counterclockwise. The following integrals were all be computed without integrating a single line integral, either by the fundemental theorem of calculus for line integrals, or by geometry, or by Green's theorem. Explain how each integral below was obtained without doing a single line integral. [Hint: start by checking if $\mathbf{F}$ or $\mathbf{G}$ is a gradient vector field.]

$$(A)\int_{C_1} \mathbf{F}\cdot\ d\mathbf{r} = 4 \qquad (B)\int_{C_1} \mathbf{G}\cdot\ d\mathbf{r} = 0 \qquad (C)\oint_{C_2} \mathbf{F}\cdot\ d\mathbf{r} = 0 \qquad (D)\oint_{C_2} \mathbf{G}\cdot\ d\mathbf{r} = 4$$

8. Sketch the region and rewrite the cylindrical triple integeral below in both spherical and rectangular coordinates, do **NOT** evaluate. (Here $\delta = \delta(x,y,z) = \delta(z,r,\theta) = \delta(\rho,\phi,\theta)$ is some density function.)

$$\int_0^{\pi/2} \int_0^1 \int_0^{r\sqrt{3}} \delta\ r\ dz\ dr\ d\theta$$

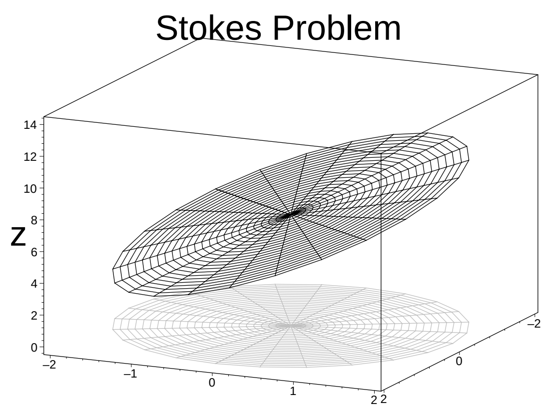
[Hint: Draw both the 2D area in the xy-plane the 3D region "lives" over, and a 2D z vs r plot.]

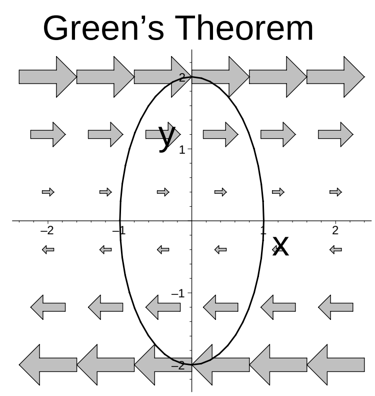
There is more test on the other side.

9. Use Stokes theorem to evaluate the line integral by doing the flux integral

$$\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{A} = \oint_{\partial S} \mathbf{F} \cdot d\mathbf{r}$$

for $\mathbf{F} = \langle y, z, x \rangle$ and S is the portion of the plane $z = f(x.y) = 2x + 3y + 7$ over the the circle $x^2 + y^2 \leq 4$ oriented upward. [Warning, note that S is not a circular disc, it is elliptically shaped.]
[You **DON'T** have to do the line integral directly.]





10. Let C be the ellipse given by $\mathbf{r}(t) = \langle \cos t, 2\sin t \rangle$ in the plane for $0 \leq t \leq 2\pi$ and let $\mathbf{F} = \langle y, 0 \rangle$ (above right). Carefully write and simplify the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ til it is a simple (non-vector) integral in the parameter t and evaluate. Then use Green's Theorem to rewrite the line integral as a double integral over the region D of points where $x^2 + y^2/4 \leq 1$. Use your TI-89 to evaluate the outer of the iterated integrals. [The inner iterated integral is easy.]