Directions: Show ALL work for credit; Give EXACT answers when possible; Start each problem on a SEPARATE page; Use only ONE side of each page; Be neat; Leave margins on the left and top for the STAPLE; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Let \( P(-2, 2, 0) \), \( Q(1, 3, -1) \) and \( R(-4, 2, 1) \). Find the equation of the plane \( S \) through the points \( P \), \( Q \) and \( R \) and the area of the triangle \( \triangle PQR \).

We have \( \overrightarrow{PQ} = (3, 1, -1) \) and \( \overrightarrow{PR} = (-2, 0, 1) \) so \( \vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ -2 & 0 & 1 \end{vmatrix} = (1, -1, 2) \).

Check: \( \overrightarrow{PQ} \cdot \vec{n} = 3 - 1 - 2 = 0 \)  
Check: \( \overrightarrow{PR} \cdot \vec{n} = -2 + 0 + 2 = 0 \)

The equation is \( x - y + 2z = 1(-2) - 1(2) + 2(0) = -4 \).

Check \( Q \): \( 1(1) - 1(3) + 2(-1) = 1 - 3 - 2 = -4 \)  
Check \( R \): \( 1(-4) - 1(2) + 2(1) = -4 - 2 + 2 = -4 \)

The area of the \( \triangle PQR = \frac{1}{2}||\vec{n}|| = \frac{1}{2}\sqrt{1+1+4} = \sqrt{6}/2. \)

2. Find the center and radius of the sphere \( S \) given by the equation \( x^2 + y^2 + z^2 - 4x + 6y - 2z = 2 \). The graph of \( S \) intersects the \( xz \)-plane in a circle, what is its equation, its center and its radius.

\[
\begin{align*}
(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 - 2z + 1) &= 2 + 4 + 9 + 1 \\
(x - 2)^2 + (y + 3)^2 + (z - 1)^2 &= 4^2
\end{align*}
\]

So the radius is 4 and the center is \( (2, -3, 1) \).

The \( xz \)-plane is \( y = 0 \) so the equation becomes

\[
\begin{align*}
(x - 2)^2 + (0 + 3)^2 + (z - 1)^2 &= 4^2 \\
(x - 2)^2 + (z - 1)^2 &= \sqrt{7}^2
\end{align*}
\]

This circle has center \( (2, 0, 1) \) and radius \( \sqrt{7} \).
3. A particle moving with speed $S$ hits a barrier at an angle of $\pi/3$ and bounces off at at an angle of $\pi/3$ in the opposite direction with the speed reduced by 20 percent. (See the figure below). Find the velocity vectors of the object both before and after impact.

Before = $S \langle \frac{1}{2}, -\sqrt{3}/2 \rangle$ and after = $S \langle \frac{2}{5}, 2\sqrt{3}/5 \rangle$.

4. Using vector operations write $\vec{a} = \langle -3, 2, 5 \rangle$ as the sum of two vectors $\vec{w} + \vec{v}$, where $\vec{w}$ is parallel to $\vec{b}$ and $\vec{v}$ is perpendicular to $\vec{b}$, when $\vec{b} = \langle -1, 0, 2 \rangle$.

Start with the unit vector in the $\vec{b}$ direction $\vec{u} = \langle -1/\sqrt{5}, 0, 2/\sqrt{5} \rangle$ $\vec{a} \cdot \vec{u} = 3/\sqrt{5} + 0 + 10/\sqrt{5} = 13/\sqrt{5}$
and hence $\vec{w} = (13/\sqrt{5})\vec{u} = \langle -13/5, 0, 26/5 \rangle$. Vector $\vec{v} = \vec{a} - \vec{w} = \langle -2/5, 2, -1/5 \rangle$

Check $\vec{v} \cdot \vec{b} = 2/5 + 0 - 2/5 = 0$.

5. Find parametric equations of the line of intersection of the two planes $x - y - z = 1$ and $11x + 5y - 5z = 20$.

Let $x = 0$ and solve $-y - z = 1$ and $5y - 5z = 20$ to get $P(0, 3/2, -5/2)$.

Let $y = 0$ and solve $x - z = 1$ and $11x - 5z = 20$ to get $Q(5/2, 0, 3/2)$.

Check $P$: $0 - 3/2 + 5/2 = 1$ and $0 + 15/2 + 25/2 = 20$.

Check $Q$: $5/2 + 0 - 3/2 = 1$ and $55/2 + 0 - 15/2 = 20$.

So $\vec{PQ} = \langle 5/2, -3/2, 4 \rangle$ and we can use $\langle 5, -3, 8 \rangle$ as the velocity vector. Thus we get $x = 5t, y = 3/2 - 3t, z = -5/2 + 8t$ for the parametric equations.