MAC 2313 Calculus 3 **Mini-Test 1** 21 Jan 2004

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Let $P(-2,2,0)$, $Q(1,3,-1)$ and $R(-4,2,1)$. Find the equation of the plane S throught the points P, Q and R and the area of the triangle $\triangle PQR$.

   We have $\overrightarrow{PQ} = \langle 3,1,-1\rangle$ and $\overrightarrow{PR} = \langle -2,0,1\rangle$ so $\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -1 \\ -2 & 0 & 1 \end{vmatrix} = \langle 1,-1,2\rangle$.

   Check: $\overrightarrow{PQ} \cdot \vec{n} = 3 - 1 - 2 = 0$ ✓
   Check: $\overrightarrow{PR} \cdot \vec{n} = -2 + 0 + 2 = 0$ ✓

   The equation is $x - y + 2z = 1(-2) - 1(2) + 2(0) = -4$.

   Check $Q:$ $1(1) - 1(3) + 2(-1) = 1 - 3 - 2 = -4$ ✓
   Check $R:$ $1(-4) - 1(2) + 2(1) = -4 - 2 + 2 = -4$ ✓

   The area of the $\triangle PQR = \frac{1}{2}\|\vec{n}\| = \frac{1}{2}\sqrt{1+1+4} = \sqrt{6}/2$.

2. Find the center and radius of the sphere S given by the equation $x^2 + y^2 + z^2 - 4x + 6y - 2z = 2$. The graph of S intersects the xz-plane in a circle, what is its equation, its center and its radius.

   $$(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 - 2z + 1) = 2 + 4 + 9 + 1$$
   $$(x-2)^2 + (y+3)^2 + (z-1)^2 = 4^2$$

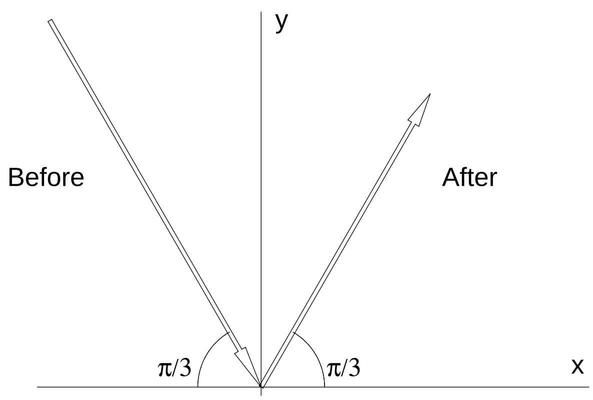
   So the radius is 4 and the center is $(2,-3,1)$.

   The xz-plane is $y = 0$ so the equation becomes

   $$(x-2)^2 + (0+3)^2 + (z-1)^2 = 4^2$$
   $$(x-2)^2 + (z-1)^2 = \sqrt{7}^2$$

   This circle has center $(2,0,1)$ and radius $\sqrt{7}$

3. A particle moving with speed S hits a barrier at an angle of $\pi/3$ and bounces off at at an angle of $\pi/3$ in the opposite direction with the speed reduced by 20 percent. (See the figure below). Find the velocity vectors of the object both before and after impact.



Before $= S\left\langle 1/2, -\sqrt{3}/2\right\rangle$ and after $= S\left\langle 2/5, 2\sqrt{3}/5\right\rangle$.

4. Using vector operations write $\vec{a} = \langle -3, 2, 5\rangle$ as the sum of two vectors $\vec{w} + \vec{v}$, where $\vec{w}$ is parallel to $\vec{b}$ and $\vec{v}$ is perpendicular to $\vec{b}$, when $\vec{b} = \langle -1, 0, 2\rangle$.

   Start with the unit vector in the $\vec{b}$ direction $\vec{u} = \left\langle -1/\sqrt{5}, 0, 2/\sqrt{5}\right\rangle$ $\vec{a}\cdot\vec{u} = 3/\sqrt{5} + 0 + 10/\sqrt{5} = 13/\sqrt{5}$ and hence $\vec{w} = (13/\sqrt{5})\vec{u} = \langle -13/5, 0, 26/5\rangle$. Vector $\vec{v} = \vec{a} - \vec{w} = \langle -2/5, 2, -1/5\rangle$

   Check $\vec{v}\cdot\vec{b} = 2/5 + 0 - 2/5 = 0\checkmark$.

5. Find parametric equations of the line of intersection of the two planes $x - y - z = 1$ and $11x + 5y - 5z = 20$.

   Let $x = 0$ and solve $-y - z = 1$ and $5y - 5z = 20$ to get $P(0, 3/2, -5/2)$.
   Let $y = 0$ and solve $x - z = 1$ and $11x - 5z = 20$ to get $Q(5/2, 0, 3/2)$.

   Check P: $0 - 3/2 + 5/2 = 1$ and $0 + 15/2 + 25/2 = 20\ \checkmark$.
   Check Q: $5/2 + 0 - 3/2 = 1$ and $55/2 + 0 - 15/2 = 20\ \checkmark$.

   So $\overrightarrow{PQ} = \langle 5/2, -3/2, 4\rangle$ and we can use $\langle 5, -3, 8\rangle$ as the velocity vector. Thus we get $x = 5t, y = 3/2 - 3t, z = -5/2 + 8t$ for the parametric equations.