MAC 2313 Calculus 3 **Mini-Test 2** 4 Feb 2004

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Show the limit below does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^3}$$

Along $y=0$ the function and the limit is 0, along $y=x$ the function and limit are $1/2$. Since $0 \neq 1/2$ the limit does not exist.

2. Consider the equation $x^2+4y^2-z^2=1$ and its graph.

   (a) Identify the graph by name.

   (b) Which of the three x-axis, y-axis or z-axis does not intersect the graph?

   (c) Carefully do a 2D plot of the $z=0$ contour of this equation (in the xy-plane).

   (d) Carefully do a 2D plot of the $y=0$ section of this equation (in the xz-plane).

   (e) Carefully do a 2D plot of the $z=\sqrt{3}$ contour of this equation (in the xy-plane).

It is a hyperboloid of one sheet. Setting $x=y=0$ yields $-z^2=1$ so the graph misses the z-axis. The $z=0$ contour is an ellispe through $(\pm1,0)$ and $(0,\pm\frac{1}{2})$. The $z=\sqrt{3}$ contour is an ellispe $(\pm2,0)$ and $(0,\pm1)$. The $y=0$ section is a hyperbola with vertices at $x=\pm1$ and missing the z-axis.

3. A particle moves at a constant speed along a line from the point $P=(1,-1,2)$ to the point $Q=(5,3,0)$. Find the parametric equation of the line if

   (a) It takes five seconds to go from P to Q.

   (b) The speed of the particle is 5 units per second.

The vector $\overrightarrow{PQ}=\langle 4,4,-2\rangle$ has length six.
The answer to (a) $x=1+\frac{4}{5}t$, $y=-1+\frac{4}{5}t$, $z=2-\frac{2}{5}t \quad 0\le t\le 5$
The answer to (b) $x=1+\frac{10}{3}t$, $y=-1+\frac{10}{3}t$, $z=2-\frac{5}{3}t \quad 0\le t\le \frac{6}{5}$

4. Consider the following equations in polar coordinates.

(I) $r=2$ (II) $r=2\sec\theta$ (III) $r=2\sin\theta$ (IV) $r=2\csc\theta$ and (V) $r=\tan\theta\sec\theta$.

Match the equations above with five of the equations in rectangular coordinates below.

(A) $y=2$	(B) $x^2=y$	(C) $x^2-2x+y^2=0$	(D) $x^2+y^2=2$
(E) $x^2+y^2=4$	(F) $y^2=x$	(G) $x=2$	(H) $x^2-2y+y^2=0$

(I) is $r^2=2^2$ or $x^2+y^2=4$ which is (E)
(II) is $r=2/\cos\theta$ or $x=2$ which is (G)
(III) is $r^2=2r\sin\theta$ or $x^2+y^2=2y$ which is (H)
(IV) is $r=2/\sin\theta$ or $y=2$ which is (A)
(V) is $r=\sin\theta/\cos^2\theta$ or $r^2\cos^2\theta=r\sin\theta$ or $x^2=y$ which is (B)

5. Consider the parametric equation $\vec{r}(t) = \langle t\cos t, t\sin t, t\rangle$ whose graph is pictured below.

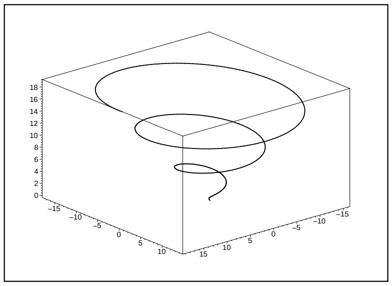
   (a) Compute the velocity (by hand).

   (b) Compute and simplify the speed (by hand). [All the trig functions will conveniently disappear.]

   (c) Use the TI-89 to find the exact arclength of the curve from $t = 0$ to $t = 1$.

   (d) Find parametric equations of the tangent line to the curve at $t = 1$.

   (e) Show each point of our curve is on the cone $x^2 + y^2 = z^2$.



Velocity is $\vec{v}(t) = \langle \cos t - t\sin t, \sin t + t\cos t, 1\rangle$. When computing the speed the middle terms of $(\cos t - t\sin t)^2$ and $(\sin t + t\cos t)^2$ cancel and using our favorite trig identity yields the speed at time t is $\sqrt{2+t^2}$. So the arclength is

$$\int_0^1 \sqrt{2+t^2}\,dt = \frac{\ln(\sqrt{3}+2)}{2} + \frac{\sqrt{3}}{2}$$

courtesy of the TI-89. At $t = 1$ the curve is at $(\cos(1), \sin(1), 1)$ and has velocity $\langle \cos(1) - \sin(1), \sin(1) + \cos(1), 1\rangle$ so the parametric equations are $x = \cos(1) + (\cos(1) - \sin(1))t, \quad y = \sin(1) + (\sin(1) + \cos(1))t, \quad z = 1 + t$. To see each point is on the cone we plug in for x, y and z.
Is $(t\cos(t))^2 + (t\sin(t))^2 = t^2$?
Is $t^2(\cos^2 t + \sin^2 t)^2 = t^2$?
Is $t^2 = t^2$? Yes