MAC 2313 Calculus 3 **Mini-Test 3** 18 Feb 2004

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

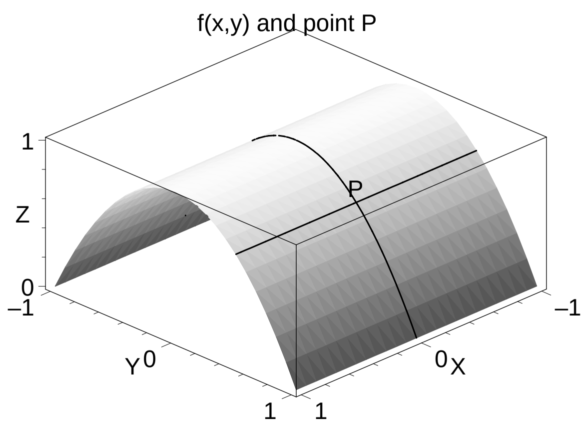
1. Find the equation of the tangent plane to $F(x, y, z) = 0$ at the point $(9, 5, 7)$ where the function $F(x, y, z)$ is given by $x^3 + 3y^2 + xz - 867$.

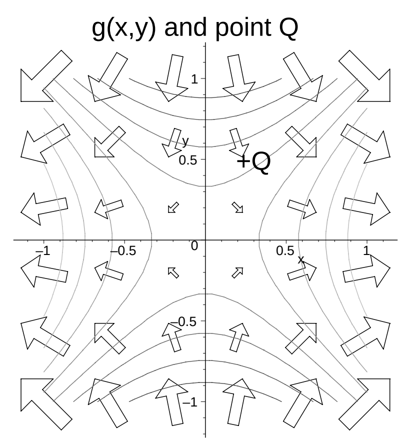
2. Use the chain rule (as shown in class) to find $\partial z/\partial u$ and $\partial z/\partial v$ when $z = (x + y)\ln(xy)$, $x = \sin(uv)$ and $y = \sqrt{u/v}$.

3. Use the pictures below to determine if the various derivatives are positive, negative or zero. The unit vector $\vec{u}$ is given by $\langle -1/\sqrt{2}, -1/\sqrt{2} \rangle$.

   (a) Use the graph (below left) to determine $f_x(P)$, $f_y(P)$, $f_{xx}(P)$, $f_{yy}(P)$ and $f_{\vec{u}}(P)$.

   (b) Use the graph (below right) to determine $g_x(Q)$, $g_y(Q)$, $g_{xx}(Q)$, $g_{yy}(Q)$ and $g_{\vec{u}}(Q)$.





4. The values of various derivatives of f at the point $(e, 1)$ have been found to be: $f(e, 1) = 5$, $f_x(e, 1) = e^{-1}$, $f_y(e, 1) = e$, $f_{xx}(e, 1) = 1 + e$, $f_{xy}(e, 1) = 1 + e^{-1}$ and $f_{yy}(e, 1) = 1 - e$.

   (a) Find $Q(x, y)$, the quadratic Taylor polynomial for f at the point $(e, 1)$.

   (b) Find parametric equations of the normal line to the surface of $z = f(x, y)$ at the point $(e, 1)$.

   (c) Find a direction $\vec{u}$ (unit vector) so that $f_{\vec{u}}(e, 1) = 0$.

5. The relative density of the earth (centered at the origin with radius 1) is given by $\sqrt{1 - \sqrt{x^2 + y^2 + z^2}}$. Find the directional derivative as you leave $P(5/26, 12/26, 0)$ heading away from the center of the earth. Do your derivatives by hand and simplify your exact answer. Use the calculator to check.