Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Find the equation of the tangent plane to $F(x, y, z) = 0$ at the point $(9, 5, 7)$ where the function $F(x, y, z)$ is given by $x^3 + 3y^2 + xz - 867$.

   Gradient is $\langle 3x^2 + z, 6y, x\rangle$ which at $(9, 5, 7)$ is $\langle 250, 30, 9\rangle$ so the equation is $250(x - 9) + 30(y - 5) + 9(z - 7) = 0$.

2. Use the chain rule (as shown in class) to find $\partial z/\partial u$ and $\partial z/\partial v$ when $z = (x + y)\ln(xy)$, $x = \sin(uv)$ and $y = \sqrt{u/v}$.

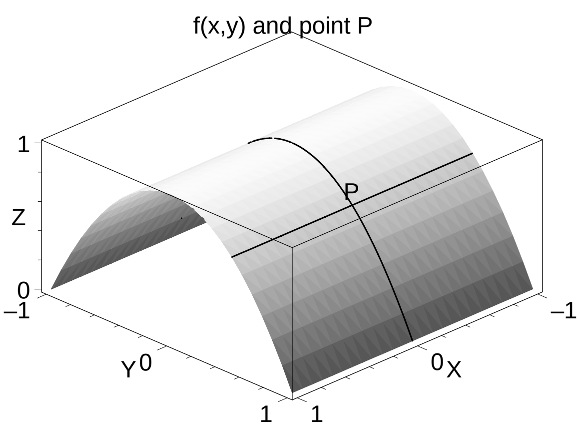
$$\frac{\partial z}{\partial u} = (1 + \frac{y}{x} + \ln(xy))(v\cos(uv)) + (1 + \frac{x}{y} + \ln(xy))(\frac{1}{2\sqrt{uv}})$$

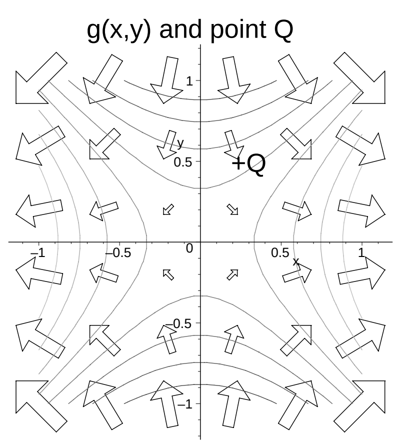
$$\frac{\partial z}{\partial v} = (1 + \frac{y}{x} + \ln(xy))(u\cos(uv)) + (1 + \frac{x}{y} + \ln(xy))(\frac{-\sqrt{u}}{2v\sqrt{v}})$$

3. Use the pictures below to determine if the various derivatives are positive, negative or zero. The unit vector $\vec{u}$ is given by $\langle -1/\sqrt{2}, -1/\sqrt{2}\rangle$.

   (a) Use the graph (below left) to determine $f_x(P)$, $f_y(P)$, $f_{xx}(P)$, $f_{yy}(P)$ and $f_{\vec{u}}(P)$.

   (b) Use the graph (below right) to determine $g_x(Q)$, $g_y(Q)$, $g_{xx}(Q)$, $g_{yy}(Q)$ and $g_{\vec{u}}(Q)$.





Answers: f_x and f_{xx} are zero, f_y, f_{yy}, g_y and g_{yy} are negative and the rest are positive.

4. The values of various derivatives of f at the point $(e, 1)$ have been found to be:
$f(e, 1) = 5$, $f_x(e, 1) = e^{-1}$, $f_y(e, 1) = e$, $f_{xx}(e, 1) = 1 + e$, $f_{xy}(e, 1) = 1 + e^{-1}$ and $f_{yy}(e, 1) = 1 - e$.

   (a) Find $Q(x, y)$, the quadratic Taylor polynomial for f at the point $(e, 1)$.
   $f(x, y) \approx 5 + e^{-1}(x - e) + e(y - 1) + ((1 + e)(x - e)^2 + 2(1 + e^{-1})(x - e)(y - 1) + (1 - e)(y - 1)^2)/2$

   (b) Find parametric equations of the normal line to the surface of $z = f(x, y)$ at the point $(e, 1)$.
   $x = e + e^{-1}t \quad y = 1 + et \quad z = 5 - t$

   (c) Find a direction $\vec{u}$ (unit vector) so that $f_{\vec{u}}(e, 1) = 0$.
   $\pm \left\langle e^2/\sqrt{e^4 + 1}, -1/\sqrt{e^4 + 1} \right\rangle$

5. The relative density of the earth (centered at the origin with radius 1) is given by $\sqrt{1 - \sqrt{x^2 + y^2 + z^2}}$. Find the directional derivative as you leave $P(5/26, 12/26, 0)$ heading away from the center of the earth. Do your derivatives by hand and simplify your exact answer. Use the calculator to check.

   The unit vector $\vec{u} = \langle 5/13, 12/13, 0\rangle$. At the point P, the value of $x^2 + y^2 + z^2 = 1/4$. The partial derivative with respect to x is $\frac{1}{2}(1 - \sqrt{x^2 + y^2 + z^2})^{-\frac{1}{2}}(-\frac{1}{2})(x^2 + y^2 + z^2)^{-\frac{1}{2}}2x$ so the gradient at P is $\left\langle -5\sqrt{2}/26, -12\sqrt{2}/26, 0 \right\rangle$ so the directional derivative is $-1/\sqrt{2}$.