1. Find the equation of the tangent plane to \( F(x, y, z) = 0 \) at the point \((9, 5, 7)\) where the function \( F(x, y, z) \) is given by \( x^3 + 3y^2 + xz - 867 \).

Gradient is \( \langle 3x^2 + z, 6y, x \rangle \) which at \((9, 5, 7)\) is \( \langle 250, 30, 9 \rangle \) so the equation is \( 250(x - 9) + 30(y - 5) + 9(z - 7) = 0 \).

2. Use the chain rule (as shown in class) to find \( \partial z/\partial u \) and \( \partial z/\partial v \) when
\[
z = (x + y) \ln(xy), \quad x = \sin(uv) \text{ and } y = \sqrt{u/v}.
\]
\[
\frac{\partial z}{\partial u} = (1 + \frac{y}{x} + \ln(xy))(v \cos(uv)) + (1 + \frac{x}{y} + \ln(xy))(\frac{1}{2\sqrt{uv}})
\]
\[
\frac{\partial z}{\partial v} = (1 + \frac{y}{x} + \ln(xy))(u \cos(uv)) + (1 + \frac{x}{y} + \ln(xy))(\frac{-\sqrt{u}}{2v\sqrt{v}})
\]

3. Use the pictures below to determine if the various derivatives are positive, negative or zero. The unit vector \( \vec{u} \) is given by \( \langle -1/\sqrt{2}, -1/\sqrt{2} \rangle \).

(a) Use the graph (below left) to determine \( f_x(P), f_y(P), f_{xx}(P), f_{yy}(P) \) and \( f_{\vec{u}}(P) \).
(b) Use the graph (below right) to determine \( g_x(Q), g_y(Q), g_{xx}(Q), g_{yy}(Q) \) and \( g_{\vec{u}}(Q) \).

Answers: \( f_x \) and \( f_{xx} \) are zero, \( f_y, f_{yy}, g_y \) and \( g_{yy} \) are negative and the rest are positive.
4. The values of various derivatives of \( f \) at the point \((e, 1)\) have been found to be:
\[
\begin{align*}
f(e, 1) &= 5, \quad f_x(e, 1) = e, \quad f_y(e, 1) = 1 + e, \quad f_{xx}(e, 1) = 1 + e^{-1} \quad \text{and} \quad f_{yy}(e, 1) = 1 - e. 
\end{align*}
\]
(a) Find \( Q(x, y) \), the quadratic Taylor polynomial for \( f \) at the point \((e, 1)\).
\[
f(x, y) \approx 5 + e^{-1}(x - e) + e(y - 1) + ((1 + e)(x - e)^2 + 2(1 + e^{-1})(x - e)(y - 1) + (1 - e)(y - 1)^2)/2
\]
(b) Find parametric equations of the normal line to the surface of \( z = f(x, y) \) at the point \((e, 1)\).
\[
x = e + e^{-1}t \quad y = 1 + et \quad z = 5 - t
\]
(c) Find a direction \( \vec{u} \) (unit vector) so that \( f_{\vec{u}}(e, 1) = 0 \).
\[
\pm \left< \frac{e^2}{\sqrt{e^4 + 1}}, -\frac{1}{\sqrt{e^4 + 1}} \right>
\]
5. The relative density of the earth (centered at the origin with radius 1) is given by \( \sqrt{1 - \sqrt{x^2 + y^2 + z^2}} \).
Find the directional derivative as you leave \( P(5/26, 12/26, 0) \) heading away from the center of the earth. Do your derivatives by hand and simplify your exact answer. Use the calculator to check.
The unit vector \( \vec{u} = \left< 5/13, 12/13, 0 \right> \). At the point \( P \), the value of \( x^2 + y^2 + z^2 = 1/4 \). The partial derivative with respect to \( x \) is \( \frac{1}{2}(1 - \sqrt{x^2 + y^2 + z^2})^{-\frac{3}{2}} \cdot (-\frac{1}{2})(x^2 + y^2 + z^2)^{\frac{1}{2}}2x \) so the gradient at \( P \) is \( \left< -5\sqrt{2}/26, -12\sqrt{2}/26, 0 \right> \) so the directional derivative is \( -1/\sqrt{2} \).