MAC 2313 Calculus 3 **Mini-Test 4** 3 Mar 2004

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Sketch the region of integration and then change the order of integration of

$$\int_0^1 \int_{\sqrt{y}}^1 f(x,y)\,dx\,dy$$

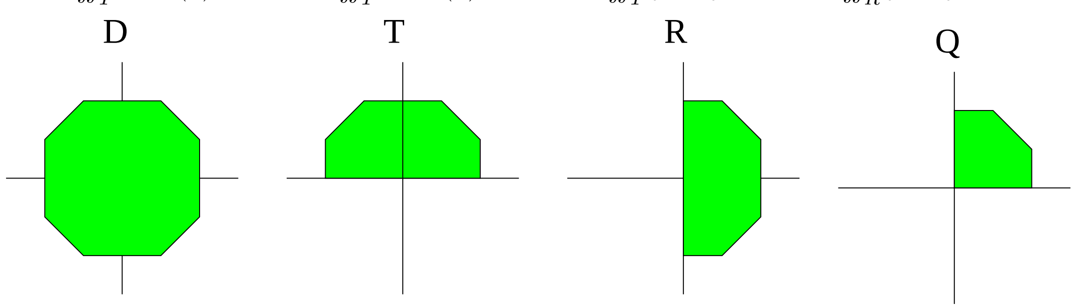
$$\int_0^1 \int_0^{x^2} f(x,y)\,dy\,dx$$

2. Let $A(0,0)$, $B(5,0)$ and $C(3,4)$. Find the coordinates of the point P in the xy-plane so the sum of the distance squares $\|\overrightarrow{PA}\|^2+\|\overrightarrow{PB}\|^2+\|\overrightarrow{PC}\|^2$ is minimum.

   The function to minimize is $f(x,y) = (x-0)^2 + (y-0)^2 + (x-5)^2 + (y-0)^2 + (x-3)^2 + (y-4)^2$. There is a global minimum, which has to be a local minimum, which has to occur at a critical point. Looking $f_x = 2x + 2(x-5) + 2(x-3) = 0$ when $3x - 8 = 0$ and $f_y = 2y + 2y + 2(y-4) = 0$ when $3y - 4 = 0$ so the only critical is at $(8/3, 4/3)$ which must be the cordinates of P.

3. The regions pictured below are inside the unit circle, decide if the following double integrals are positive, negative or zero.

   $A = \iint_D x^2 + y^2\,dA$ $B = \iint_T 5y\,dA$ $C = \iint_T 5x\,dA$ $D = \iint_R 5x\,dA$ $E = \iint_Q 0\,dA$
   $F = \iint_T -\sin(x)\,dA$ $G = \iint_T -\cos(x)\,dA$ $H = \iint_T y^3 - y\,dA$ $I = \iint_R y^3 - y\,dA$ $J = \iint_R x + y^2\,dA$



   $A+$ $B+$ $C0$ $D+$ $E0$
   $F0$ $G-$ $H-$ $I0$ $J+$

4. Use your TI-89 to find all the critical points of the function $f(x,y) = 8y^3 + 12x^2 - 24xy$, then show how you would obtain these critical points by hand. Classify these local extrema by filling out a table like the one below, with a separate line for each critical point.

(x,y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
?	?	?	?	?	?

   The calculator gives two critical points. By hand we find $f_x = 24x - 24y$ and $f_y = 24y^2 - 24x$. Setting $f_x = 0$ yields $y = x$ and plugging into $f_y = 0$ yields $y^2 - y = y(y-1) = 0$, so we have critical points at $(0,0)$ and $(1,1)$. We need $f_{xx} = 24$, $f_{yy} = 48y$, and $f_{xy} = -24$. The table becomes

(x,y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
(0,0)	24	0	-24	-576	Saddle
(1,1)	24	48	-24	576	Local Minimum

5. Use Lagrange multipliers to find the maximum and minimum **VALUES** of $f(x, y) = x^2 - y$ subject to the constraint that $x^2 + y^2 = 4$.

   Let $F(x, y, \lambda) = x^2 - y - \lambda(x^2 + y^2 - 4)$. $F_x = 2x - 2\lambda x$, $F_y = -1 - 2\lambda y$, and $F_\lambda = -(x^2 + y^2 - 4)$. Setting $F_x = 0$ yields $x(1 - \lambda) = 0$ so $x = 0$ or $\lambda = 1$. Setting $F_y = 0$ yields $y = -1/(2\lambda)$. Keying off $F_x = 0$ if $x = 0$, then plugging in $x^2 + y^2 = 4$ yields $y = \pm 2$ and two points to check. On the other hand if $\lambda = 1$, then the f_y equation says $y = -1/2$. Plugging this y into the constrain yields $x = \pm\sqrt{15}/2$. Checking values:
   $f(0, 2) = -2$
   $f(0, -2) = 2$
   $f(\sqrt{15}/2, -1/2) = 17/4$
   $f(-\sqrt{15}/2, -1/2) = 17/4$
   so $17/4$ is the maximum value and -2 is the minimum value.