MAC 2313 Calculus 3 **Mini-Test 6** 14 Apr 2004

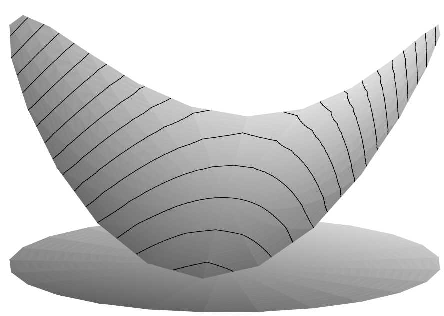
**Directions:** Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Compute div $\langle x\ln(x+y), x\ln(xy), \ln(x/z)\rangle$ and curl $\langle xyz, x^2y^2z^2, 0\rangle$

2. Let $C$ be the line from $(0,1,0)$ to $(5,1,-1)$ and let $\vec{F} = \langle z, z^2+3y^2, x+2yz+2z\rangle$ compute the line integral $\int_C \vec{F}\cdot d\vec{r}$ two ways. First do it directly, and secondly do it using the fundamental theorem of line integrals.

3. Are the following statements true or false?
   (a) The curl $\vec{F}$ is a scalar field.
   (b) The div $\vec{F}$ is a scalar field.
   (c) The $\nabla\vec{F}$ is a vector field.
   (d) A vector field $\vec{F}$ is a gradient vector field if its diverence is zero.
   (e) $\int_C \vec{F}\cdot d\vec{r} = \vec{F}(Q) - \vec{F}(P)$ where $P$ and $Q$ are the endpoints of $C$.
   (f) If $\vec{F}$ is path independent, then $\oint_C \vec{F}\cdot d\vec{r} = 0$ for any closed curve $C$.
   (g) If $\vec{F} = \langle -y, x\rangle$ and $C$ is a closed curve oriented counterclockwise which is $\partial D$ for some region $D$ in the plane, then $\oint_C \vec{F}\cdot d\vec{r} = 2\,\text{area}(D)$.
   (h) The vector field $\vec{F} = \vec{r}$ in 3-space is path independent.
   (i) If $S_1$ is a rectangle with area 1 and $S_2$ is a rectangle with area 2, then $2\iint_{S_1}\vec{F}\cdot d\vec{A} = \iint_{S_2}\vec{F}\cdot d\vec{A}$.
   (j) If $\vec{F} = 2\vec{G}$, then $\iint_S \vec{F}\cdot d\vec{A} = 2\iint_S \vec{G}\cdot d\vec{A}$.

4. Compute the flux integral of $\vec{F} = \langle x, y, z\rangle$ over the surface $S$ (below) which is the portion of $z = f(x,y) = 10 + x^2 + y$ over the circle of radius 5 centered at the origin and is oriented upward.



5. Check Gauss's Theorem by computing both integrals below. Let $H$ be the the half of the ball of radius 2 where $z \geq 0$ and let $S = \partial H$ be the outward-oriented surface which is the union of a hemisphere $T$ on top and a flat disk $B$ on bottom. Use the vector field $\vec{F} = \langle 0, 0, 1+z\rangle$.

$$\iiint_H \text{div}\vec{F}\,dV = \iint_S \vec{F}\cdot d\vec{A}$$