Practice Mini-Test 1 – Calculus 3 – Spring 04

1. Q3 F03 Plot the points $P(3, 5, -1)$ and $Q(-3, 3, 5)$ on a 3D graph (whose axeses are in the usual positions). Draw the vector $\overrightarrow{PQ}$ on the graph and write $\overrightarrow{PQ}$ in the $\langle ?, ?, ? \rangle$ notation.

2. T1#4 S02 Find the center and radius of the sphere S given by the equation $x^2+y^2+z^2+2x+8y-4z = 28$. The graph of S intersects the xz-plane in a circle, what is its equation, its center and its radius. [compare T1#1 F03]

3. T1#1 S03 Find the equation of the plane parallel to the plane $3x - 4y - 6z = 21$ and passing through the point $(-3, 1, 2)$ and find the distance between the two parallel planes. [compare T1#1 F02]

4. T1#2 F03 Find the equation of the plane through the points $(2, 1, -2), (3, -1, 2)$ and $(4, 0, 1)$. [compare T1#2 S03, T1#2 F02]

5. T1#3 F03 Let $P(3, -2, 2)$ and $\vec{v} = \langle 3, -1, 5 \rangle$, find:
   (a) The equation of the line through P in the direction of $\vec{v}$
   (b) The coordinates of the point where the line in (a) intersects the xz-plane.
   (c) The equation of the plane perpendicular to $\vec{v}$ through P.
   (d) The coordinates of the point where the y-axis intersects the plane in (c).

6. T1#6 F03 A treasure map reads start at the big X, walk 40 paces north, 20 paces northwest and dig a hole 10 paces deep. Write the vector $\vec{v}$ that goes from the big X to the bottom of the hole and find the exact simplified value of the length squared $||\vec{v}||^2$. (The x-axis points East, the y-axis points North, and the z-axis points up.) [compare T1#3 S02, T1#6 F02]

7. T1#8 F03 Using vector operations write $\vec{a} = \langle 2, -1, 5 \rangle$ as the sum of two vectors, one parallel (say $\vec{v}$), and one perpendicular (say $\vec{w}$) to $\vec{b} = \langle -4, 4, 2 \rangle$. [compare T1#8 S03, T1#8 F02]

8. T1#6 S03 Determine if the lines L_1 and L_2 are parallel, skew or intersecting. If they intersect, find the point of intersection.
   $L_1: \quad x = 2 + t, y = 2 - t, z = 5 + 3t$
   $L_2: \quad x = 1 - s, y = 1 + 2s, z = -6 + s$ [compare T1#4 F02]

9. T1#7 S03 Find the parametric equation of the line through the points $P(3, 2, 8)$ and $Q(4, 4, -4)$ and find the two points where it it intersects the elliptical paraboloid $z = x^2 + y^2$. [compare T1#10 S02]

10. T1#9 F03 Find parametric equations of the line of intesection of the two planes $x + 2y + 2z = 3$ and $3x + 2y - 2z = 9$.

11. T1#3 S02 For the given vector, write it as an expression in terms of the vectors $\vec{a}$ and $\vec{b}$ suggested by the picture below.
    (a) $\vec{x}$
    (b) $\vec{w}$
    (c) $\vec{y}$
    (d) the unit vector $\vec{u}$
    (e) $\vec{v}$