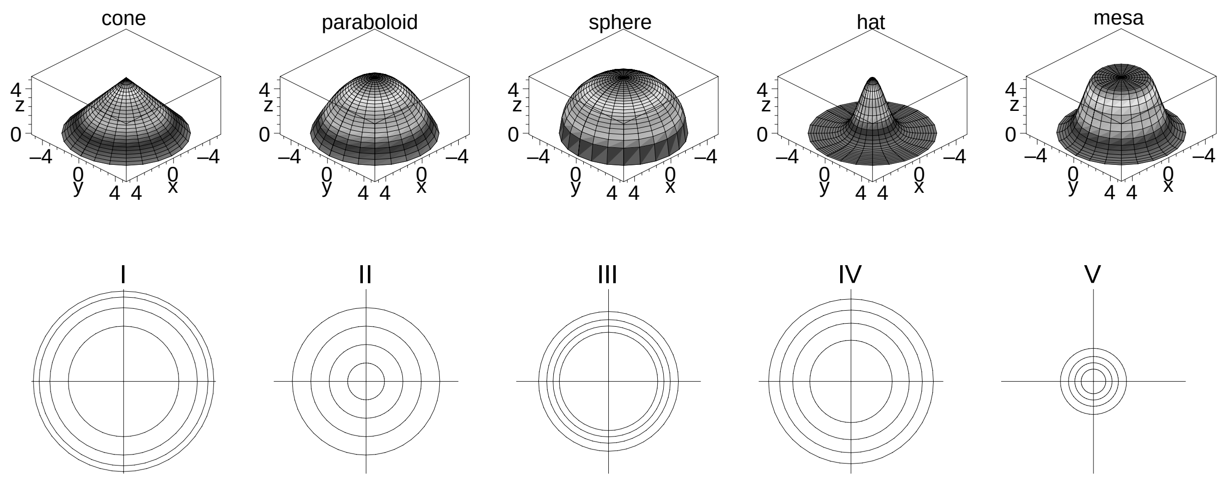
Practice Mini-Test 2 – Calculus 3 – Spring 04

1. [Intersections of parametric curves with surfaces (last mini-test did straight lines) and the intersection of two parametric curves (homework and last mini-test did straight lines).] Find the points where the helix $\vec{r}(t) = \langle 3\cos t, 3\sin t, 4t\rangle$ intersects the sphere $x^2 + y^2 + z^2 = 5^2$. [compare PMT1 #8 S04]

2. T2#1 S03 Show the limit below does not exit.

$$\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$$

3. T1#5 S03 Match the plot3ds to the contourplots. Each contourplot plots the four contours $z = 1, 2, 3$ and 4 and each 3d plot is over the disk $x^2 + y^2 \le 25$.



[Compare T1#4 F03 T1#3 F02]

4. T1#5 F03 Make sure your TI-89 is in radian mode and use it to do parts (a) & (b) but you must do (c) and (d) by hand.

   (a) Find the exact answer to $\int \sin^4 t \cos^3 t \, dt$.

   (b) Plot the curve $\vec{r}(t) = \left\langle \cos^4 t, \sin^4 t \right\rangle$ for $0 \le t \le \pi/2$

   (c) Find the velocity $\vec{v}(t)$ of the curve in (b).

   (d) Find the acceleration $\vec{a}(t)$ of the curve in (b). and acceleration. [Compare T1#9 S03 T1#7 F02]

5. T1#7 F03 Polar coordinates.

   (a) Convert the polar equation $r = 2\cos\theta + 4\sin\theta$ to rectangular (Cartesian) coordinates and COMPLETELY identify the curve and sketch it.

   (b) Convert $1 = x^2 y + y^3$ to polar coordinates, and solve for r. Simplify. [hint: the answer is some trig function to some power.]

6. T1#10 F02 Plot the contour lines for the equation $x^2 + (y - z)^2 = 4$ for the z values $z = 0$, 1, 2, and 3. Label your contours with the z values. [Since the equation is not a function $z = f(x, y)$, it is ok for the contours to intersect.] On a separate graph give a 3D sketch of the surface and describe the graph in words.

7. T1#10 S03 Catalog of functions. For questions below list all of equations (a)–(b) (see below) that satisfy the given condition, if there are none that satisfy condition then say "none". [Hint: Sometimes it is easier to say all but "these", then to list the ones that do.]

   (a) Which are hyperboloids?

   (b) Which are cylinders?

   (c) Which contain the origin?

   (d) Which are unbounded?

   (e) Which intersect the y-axis?

   The list of equations: (Same as the list on the homework problem T1#10 S00)

   (a) $x^2 + 4y^2 + 9z^2 = 1$

   (b) $9x^2 + 4y^2 + z^2 = 1$

   (c) $x^2 - y^2 + z^2 = 1$

   (d) $-x^2 + y^2 - z^2 = 1$

   (e) $y^2 = 2x^2 + z^2$

   (f) $y = x^2 + 2z^2$

   (g) $x^2 + 2z^2 = 1$

   (h) $y = x^2 - z^2$

   [Compare T1#6 S02 T1#10 S00]

8. T3#10 S00 Consider the parametric equations for $0 \le t \le \pi$.

   (I) $\langle \cos(2t), \sin(2t) \rangle$ (II) $\langle 2\cos(t), 2\sin(t) \rangle$ (III) $\langle \cos(t/2), \sin(t/2) \rangle$ and (IV) $\langle 2\cos(t), -2\sin(t) \rangle$.

   (a) Match the equations above with four of the curves A, B, C, D, E and F in graph below.

   (b) Give parametric equations for the curves which have not been matched, again assuming $0 \le t \le \pi$.