Practice Mini-Test 2 – Calculus 3 – Spring 04

1. [Intersections of parametric curves with surfaces (last mini-test did straight lines) and the intersection of two parametric curves (homework and last mini-test did straight lines).] Find the points where the helix $\vec{r}(t) = \langle 3\cos t, 3\sin t, 4t\rangle$ intersects the sphere $x^2 + y^2 + z^2 = 5^2$. [compare PMT1 #8 S04]

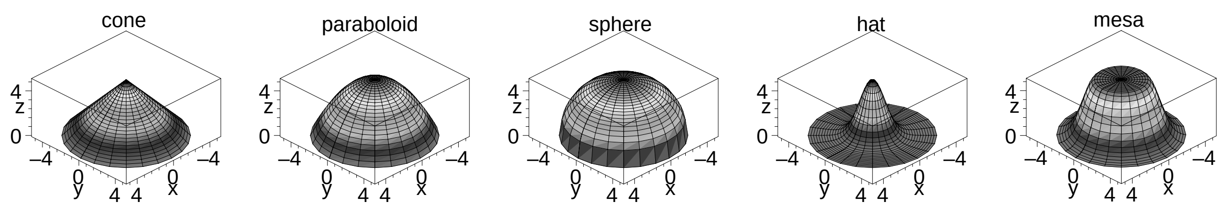
   Plug the helix equations into the sphere $(3\cos t)^2 + (3\sin t)^2 + (4t)^2 = 25$ and use $\cos^2 t + \sin^2 t = 1$ to obtain $9 + 16t^2 = 25$ or $t = \pm 1$. This gives the points as $(3\cos(1), 3\sin(1), 4)$ when $t = 1$ and $(3\cos(1), -3\sin(1), -4)$ when $t = -1$. Note we have simplified the negative signs inside the trig function using the fact $\sin t$ is odd and $\cos t$ is even.

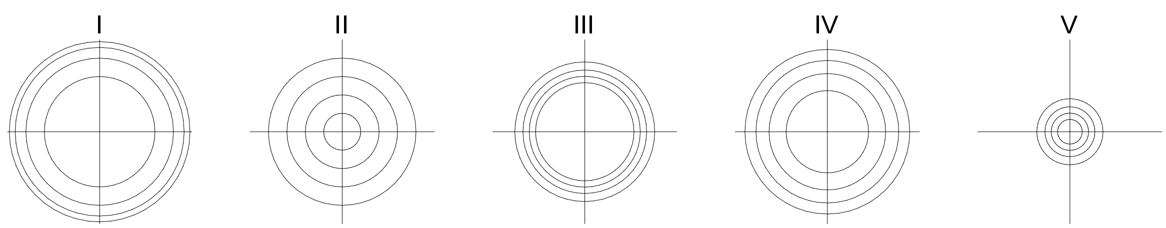
2. T2#1 S03 Show the limit below does not exit.

$$\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2 + y^2}$$

   The limit along the x-axis, where $y = 0$, is $\lim_{x\to 0} 0^2/(x^2 + 0^2) = 0$ and the limit along the y-axis, where $x = 0$, is $\lim_{y\to 0} y^2/(0^2 + y^2) = 1$. Since $0 \neq 1$ the limit does not exist. There are other paths that work, along $y = x$ the limit is $1/2$, and infinitely many other possible directions.

3. T1#5 S03 Match the plot3ds to the contourplots. Each contourplot plots the four contours $z = 1, 2, 3$ and 4 and each 3d plot is over the disk $x^2 + y^2 \leq 25$.





   [Compare T1#4 F03 T1#3 F02]

   Usually these kind of problems divide into "halves". The hat and mesa are "different" from the other three. They have steeper sides which don't start at the boundary. The hat has the contours nearer the center so hat is V and mesa is III. That leaves 3 more. The cone has uniform growth so its the contours are uniformly spaced so the cone is II (which might be the easiest of the five). The sphere increases faster at outer circle than does the paraboloid, so the sphere is I and the paraboloid is IV.

4. T1#5 F03 Make sure your TI-89 is in radian mode and use it to do parts (a) & (b) but you must do (c) and (d) by hand.

   (a) Find the exact answer to $\int \sin^4 t \cos^3 t \, dt$.
   Using the TI-89 the key sequence is f3 2 to pick integral, then the integrand, then comma t and). Finally copy the answer and add $+C$. The TI-89 produces $\frac{-\sin(t)((5\sin^2(t)+3)\cos^4(t)-\cos^2(t)-2)}{35}+C$ but the simpler answer $\frac{1}{5}\sin^5 t + \frac{1}{7}\sin^7 t + C$ can obtained using calculus 2 methods.

   (b) Plot the curve $\vec{r}(t) = \langle \cos^4 t, \sin^4 t \rangle$ for $0 \le t \le \pi/2$
   Sort of what happens when a straight line from $(0,1)$ to $(1,0)$ is pulled towards the origin "smoothly".

   (c) Find the velocity $\vec{v}(t)$ of the curve in (b).
   Velocity is $\langle -4\cos^3 t \sin t, 4\sin^3 t \cos t \rangle$ by the chain rule.

   (d) Find the acceleration $\vec{a}(t)$ of the curve in (b). and acceleration.
   The acceleration is $\langle 12\cos^2 t \sin^2 t - 4\cos^4 t, 12\sin^2 t \cos^2 t - 4\sin^4 t \rangle$ which uses both the product rule and the chain rule.

   [Compare T1#9 S03 T1#7 F02]

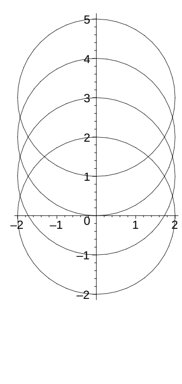
5. T1#7 F03 Polar coordinates.

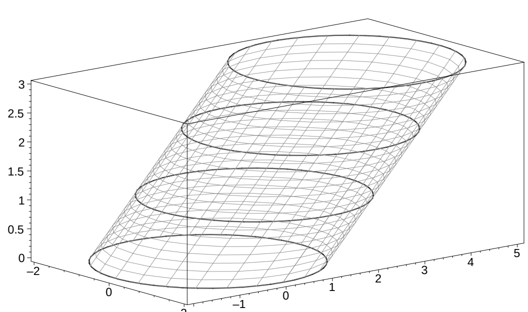
   (a) Convert the polar equation $r = 2\cos\theta + 4\sin\theta$ to rectangular (Cartesian) coordinates and COMPLETELY identify the curve and sketch it.
   Multiply by r obtaining $r^2 = 2r\cos\theta + 4r\sin\theta$ or $x^2 + y^2 = 2x + 4y$. Complete the square to obtain $(x-1)^2 + (y-2)^2 = 1 + 4 = (\sqrt{5})^2$. We have a circle centered at $(1,2)$ of radius $\sqrt{5}$. Don't forget to sketch the circle. [Does it contain the origin? intersect with either axis?]

   (b) Convert $1 = x^2 y + y^3$ to polar coordinates, and solve for r. Simplify. [hint: the answer is some trig function to some power.]
   $1 = (r\cos\theta)^2 r\sin\theta + (r\sin\theta)^3 = r^3 \sin\theta(\cos^2\theta + \sin^2\theta) = r^3\sin\theta$ and solving for r we get $r = (\csc\theta)^{1/3}$.

6. T1#10 F02 Plot the contour lines for the equation $x^2 + (y-z)^2 = 4$ for the z values $z = 0$, 1, 2, and 3. Label your contours with the z values. [Since the equation is not a function $z = f(x,y)$, it is ok for the contours to intersect.] On a separate graph give a 3D sketch of the surface and describe the graph in words.

   The contour for $z = k$ is a circle centered at $(0,k)$ of radius 2. The circles overlap, the surface is an "oblique" circular cylinder as it leans in the y direction as z increases.





7. T1#10 S03 Catalog of functions. For questions below list all of equations (a)–(h) (see below) that satisfy the given condition, if there are none that satisfy condition then say "none". [Hint: Sometimes it is easier to say all but "these", then to list the ones that do.]

   (a) Which are hyperboloids? Both (c) and (d).

   (b) Which are cylinders? Only (g).

   (c) Which contain the origin? We have (e), (f) and (h).

   (d) Which are unbounded? All but the ellipsoids (a) and (b).

   (e) Which intersect the y-axis? All but (c) and (g).

   The list of equations: (Same as the list on the homework problem T1#10 S00)

   (a) $x^2 + 4y^2 + 9z^2 = 1$
   Ellipsoid, bounded, $(0, 0, 0)$ is a not solution, and $(0, \pm 1/2, 0)$ is the intersection with the y-axis.

   (b) $9x^2 + 4y^2 + z^2 = 1$
   Ellipsoid, bounded, $(0, 0, 0)$ is a not solution, and $(0, \pm 1/2, 0)$ is the intersection with the y-axis.

   (c) $x^2 - y^2 + z^2 = 1$
   Hyperboloid of one sheet, unbounded, $(0, 0, 0)$ is a not solution, and no $(0, y, 0)$ is a solution so it misses the y-axis.

   (d) $-x^2 + y^2 - z^2 = 1$
   Hyperboloid of two sheets, unbounded, $(0, 0, 0)$ is a not solution, and $(0, \pm 1, 0)$ is the intersection with the y-axis.

   (e) $y^2 = 2x^2 + z^2$
   Elliptic cone, unbounded, $(0, 0, 0)$ is a solution, and $(0, 0, 0)$ is the intersection with the y-axis.

   (f) $y = x^2 + 2z^2$
   Elliptic paraboloid, unbounded, $(0, 0, 0)$ is a solution, and $(0, 0, 0)$ is the intersection with the y-axis.

   (g) $x^2 + 2z^2 = 1$
   Elliptic cylinder, unbounded, $(0, 0, 0)$ is not a solution, and no $(0, y, 0)$ is a solution so it misses the y-axis.

   (h) $y = x^2 - z^2$
   Hyperbolic paraboloid, unbounded, $(0, 0, 0)$ is a solution, and $(0, 0, 0)$ is the intersection with the y-axis.

   [Compare T1#6 S02 T1#10 S00]

8. T3#10 S00 Consider the parametric equations for $0 \le t \le \pi$.

(I) $\langle \cos(2t), \sin(2t) \rangle$ Radius 1 starts at $(1, 0)$ ends at $(1, 0)$ [counterclockwise] it is D.
(II) $\langle 2\cos(t), 2\sin(t) \rangle$ Radius 2 starts at $(2, 0)$ ends at $(-2, 0)$ [counterclockwise] it is A.
(III) $\langle \cos(t/2), \sin(t/2) \rangle$ Radius 1 starts at $(1, 0)$ ends at $(0, 1)$ [counterclockwise] it is B.
and (IV) $\langle 2\cos(t), -2\sin(t) \rangle$. Radius 2 starts at $(2, 0)$ ends at $(-2, 0)$ [clockwise] it is F.

(a) Match the equations above with four of the curves A, B, C, D, E and F in graph below.

(b) Give parametric equations for the curves which have not been matched, again assuming $0 \le t \le \pi$. We have C and E left. So C is like F with radius 1/2 which can be written as $\left\langle \frac{1}{2}\cos(t), -\frac{1}{2}\sin(t) \right\rangle$. While E can be written as "minus 2" B or $\langle -2\cos(t/2), -2\sin(t/2) \rangle$. There are infinitely many other correct answers.