Practice Mini-Test 3 – Calculus 3 – Spring 04

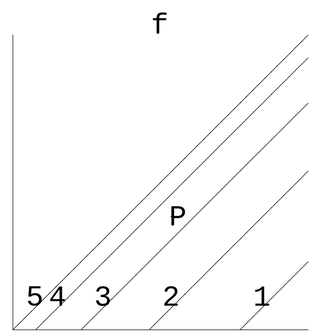
1. T2#1 F03 Chain Rule. [Compare T2#2 S03 T2#1 F02] Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$ when $z = \tan(x^2 - y^2), x = s/t$ and $y = \sqrt{t}$.

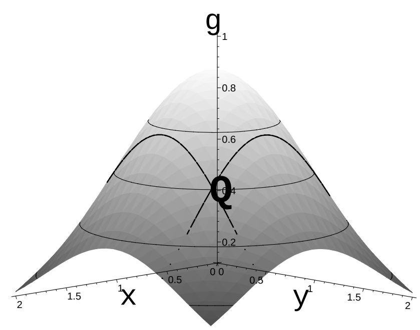
2. T2#6 F03 Tangent Plane. [Compare T2#6 S03 T2#7 F02] Find the equation of the tangent plane to the level surface $F(x, y, z) = x^2 + 2y^2 + 6xy - 8x^3z + 27 = 0$ at $(1, -2, 3)$.

3. T2#4 F03 Directional Derivative. [Compare T2#4 S03 T2#5 F02] Find the directional derivative of $f(x, y, z) = x^3 + y^2 + z$ as you leave the point $P(3, 2, 1)$ heading in the direction of the point $Q(0, 6, 1)$.

4. T2#6 F02 Positive, Negative or Zero. [Compare T2#5 S03 T2#5 F03] The point P is on the contour graph of the function f (below left) and the point Q is on the surface of the graph of the function g (below right). Let $\vec{u}$ be the unit vector $\vec{u} = (-\vec{i} - \vec{j})/\sqrt{2}$.

   Find the sign (positive, negative or zero) of the partials of f: $f_x(P), f_y(P), f_{xx}(P), f_{yy}(P), f_{xy}(P)$ and the partials of g: $g_x(Q), g_y(Q), g_{xx}(Q)$ and the two directional derivatives $f_{\vec{u}}(P)$ and $g_{\vec{u}}(Q)$.



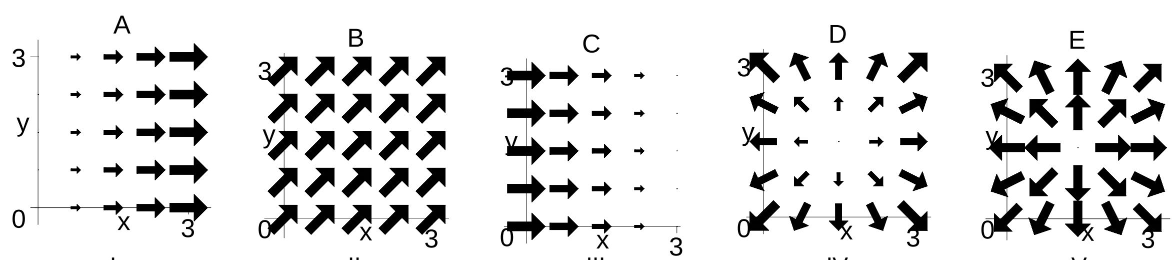


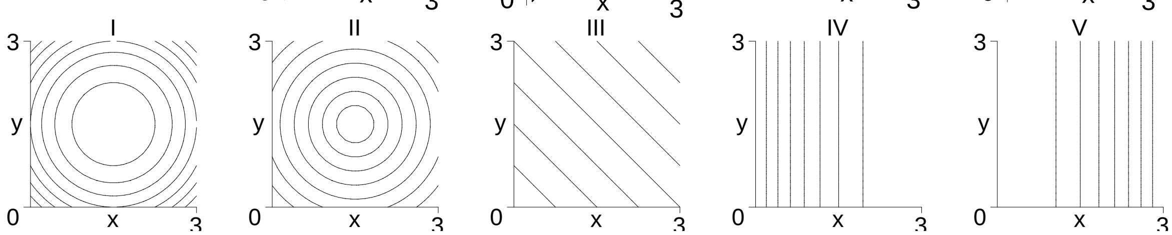
5. T2#3 F02 Taylor/Normal. For the function $f(x, y) = x^3 + xy + y^2$

   (a) Compute the quadratic Taylor polynomial for f at the point $(-1, 2)$.

   (b) Compute the equation of the normal line to f at the point $(-1, 2)$.

6. T2#3 F03 Gradient plots matching. Match each of the gradient plots A–E to the matching contour plot among I–V.





7. T2#6 S03 2D/3D tangents (This is problem #30 from H14.5). A differentiable function $f(x, y)$ has the property that $f(1, 2) = 7$ and $\nabla f(1, 2) = 2\vec{\mathbf{i}} - 5\vec{\mathbf{j}}$.

   (a) Find the equation of the tangent PLANE to the SURFACE $z = f(x, y)$ at the point $(1, 2, 7)$.

   (b) Find the equation of the tangent LINE to the level CURVE of f through the point $(1, 2)$.

8. T2#2 F00 Directional derivative application (also a problem from the text (second edition)). Suppose that $F(x, y, z) = x^2 + y^4 + x^2z^2$ gives the concentration of salt in a fluid at the point (x, y, z) and you are at the point $(-1, 1, 1)$.

   (a) In which direction (unit vector) should you move if you want the concentration to increase the fastest?

   (b) Suppose you start to move in the direction you found in part (a) at a speed of 4 units/sec. How fast is the concentration changing?