Practice Mini-Test 4 – Calculus 3 – Spring 04

1. T2#7 F03 Classify local extrema. [Compare T2#9 S03 T2#9 F02 T2#10 S02] Use your TI-89 to find all the critical points of the function $f(x, y) = x^3 - 3xy + y^3$, then show how you would obtain these critical points by hand. Classify these local extrema by filling out a table like the one below, with a separate line for each critical point.

(x, y)	f_{xx}	f_{yy}	f_{xy}	big D	Classification
?	?	?	?	?	?

2. T2#10 F02 Lagrange multipliers. [Compare T2#10 F03 T2#8 S03] Use your TI-89 to plot the $z = 1$ contour of the function $z = g(x, y) = x^2 + xy + y^2$. On the same graph, plot some contour lines for $f(x, y) = x + y$. Use Lagrange Multipliers to find the maximum and minimum **VALUES** for $f(x, y)$ on the constraint $g(x, y) = 1$.

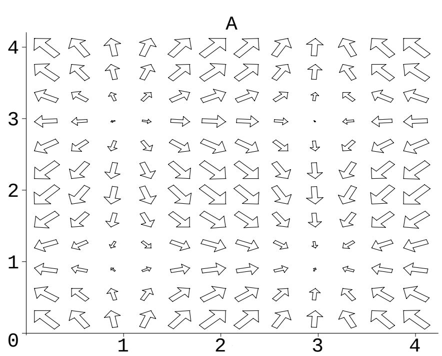
3. T3#4 S03 Change the order [Compare T2#8 F02 T3#6 S02] Sketch the region of integration and reverse the order of integration for

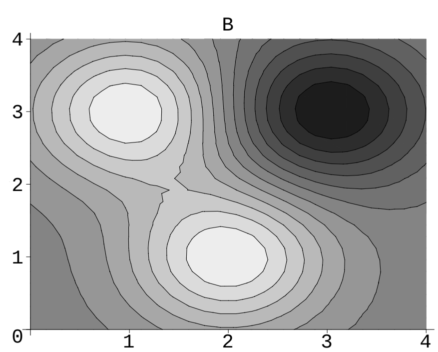
$$\int_0^2 \int_{x^2}^{2x} dy\ dx$$

[You do **NOT** have to evaluate the integrals, but using the TI-89 to evaluate both integrals would be a way of checking your answer.]

4. T2#10 S03 Optimization. [Compare T2#9 F96] Find the coordinates of the **POINT** closest to the point $P(1, 1, 0)$ which is both on the surface $z^2 = x^2 + y^2$ AND is in the first octant.

5. T2#4 F02 Local Extremes from graphs [Compare T2#9c S02 T2#5] The graph A is a plot of ∇f, the gradient of f and the graph B is a contourplot of g. (Light regions have higher values than dark regions.] Find the co-ordinates of all extrema of f and g and **LABEL** them as either local minimums, local maximums or saddle points.





6. Positive, Negative or Zero. Let $D = \{(x, y) : x^2 + y^2 \leq 1\}$, $T = \{(x, y) : y \geq 0, x^2 + y^2 \leq 1\}$, $R = \{(x, y) : x \geq 0, x^2 + y^2 \leq 1\}$ and $Q = \{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$. Determine if the following are positive negative or zero.

$\iint_D x\,dA$	$\iint_T x\,dA$	$\iint_R x\,dA$	$\iint_Q x\,dA$
$\iint_D x^2 + y^2\,dA$	$\iint_T x^2 + y^2\,dA$	$\iint_R x^2 + y^2\,dA$	$\iint_Q x^2 + y^2\,dA$
$\iint_D \sin y\,dA$	$\iint_T \sin y\,dA$	$\iint_R \sin y\,dA$	$\iint_Q \sin y\,dA$
$\iint_D xy\,dA$	$\iint_T xy\,dA$	$\iint_R xy\,dA$	$\iint_Q xy\,dA$

7. T2#6 F00 Graphs from local extrema. The function $f(x, y)$ has local maximums at $(1, 0)$ and $(-1, 0)$, local minimums at $(0, 1)$ and $(0, -1)$ and one saddle point at the orgin.

   (a) Sketch a possible contour graph for f.

   (b) Sketch a possible graph for ∇f.