Practice Mini-Test 5 – Calculus 3 – Spring 04

1. T2#8 F03 Rect/Cyl/Sphere [Compare T2#7 S03 T3#8 F02 T3#8 F00 T3#3 S00] Sketch the region and rewrite the cylinderical triple integral below in both spherical and rectangular coordinates, do NOT evaluate. (Here $\delta = \delta(x, y, z) = \delta(z, r, \theta) = \delta(\rho, \phi, \theta)$ is some density funtion.)

$$\int_0^{\pi/2} \int_0^2 \int_{-\sqrt{4-r^2}}^0 \delta \, r \, dz \, dr \, d\theta$$

The trick with these to draw two graphs, the shadow in the xy-plane and the zr-crosssection which is a half plane since $r \geq 0$. The outer two integrals say the xy shadow is the portion of the disk of radius 2 in the first quadrant. The innermost integral is also a quarter circle, but this time in the fourth quadrant of zr-crosssection. Rectangular followed by spherical:

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^0 \delta \, dz \, dy \, dx \qquad \int_0^{\pi/2} \int_{\pi/2}^{\pi} \int_0^2 \delta \, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

2. T2#9 F03 Polar [Compare T3#3 F00] Sketch the region of integration of the polar coordinate integral

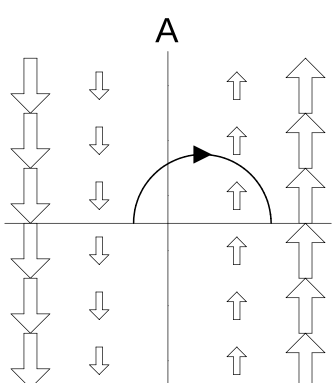
$$\int_0^{\pi/2} \int_0^2 r^2 \cos\theta \, dr \, d\theta$$

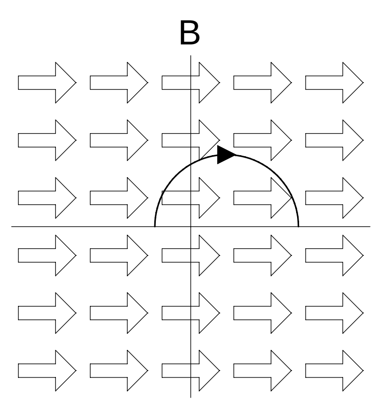
and rewrite it as a rectangular integral (or sum of integrals) in BOTH orders $dx\,dy$ and $dy\,dx$ and evaluate all the integrals on your TI-89.

Again a quarter circle in the first quadrant of radius 2. One of the r in $r^2 \cos\theta$ is part of the dA, so the integrand is x in rectangle land. The integrals are:

$$\int_0^2 \int_0^{\sqrt{4-x^2}} x \, dy \, dx \qquad \int_0^2 \int_0^{\sqrt{4-y^2}} x \, dx \, dy$$

3. T3#4 F03 Vector fields [Compare T3#6 F02 T3#7 S02 T3#6 F00] Find formulas for the two vector fields below. (There are many possible answers.) Decide if the line integrals over the given curves will be positive negative or zero in each plot.





A is $\langle 0, x \rangle$ and the line integral is negative. B is $\langle 1, 0 \rangle$ and the line integral is positive

4. T3#3 S03 Volume. Write down a triple integral which will give the volume of the region under the surface $z = 1 + x + y^2$ above the triangle $T = \{(x, y) : 0 \leq y \leq 1, 0 \leq x \leq y\}$ in the xy-plane. Do NOT evaluate.

$$\int_0^1 \int_x^1 \int_0^{1+x+y^2} 1 \, dz \, dy \, dx$$

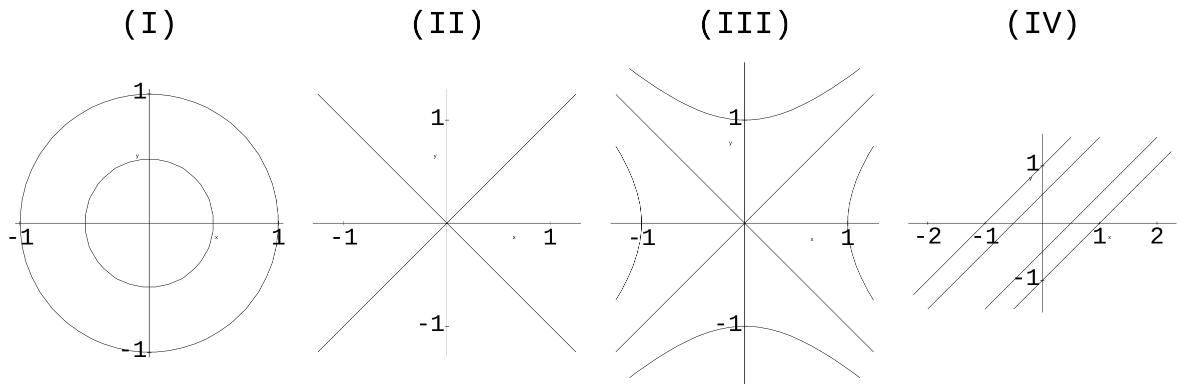
5. T3#4 S02 Mass. Write down triple integrals in each of the coordinate systems spherical, cylinderical and rectangular which will compute the mass of the hemisphere region given by $x^2+y^2+z^2 \leq R^2$ and $z \geq 0$ with density function δ. Do NOT evaluate.

$$sphere = \int_0^{2\pi} \int_0^{\pi/2} \int_0^R \delta \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$cylinder = \int_0^{2\pi} \int_0^{2} \int_0^{\sqrt{R^2-r^2}} \delta r \, dz \, dr \, d\theta$$

$$rectangle = \int_{-2}^{2} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \int_0^{\sqrt{R^2-x^2-y^2}} \delta \, dz \, dy \, dx$$

6. T3#6 S00 Flows Match the following vector fields with their flow lines. (a)$\langle y, x\rangle$ (b)$\langle -y, x\rangle$ (c)$\langle x, y\rangle$ (d)$\langle x-y, x-y\rangle$.



The circles (I) are from $\langle -y, x\rangle$. The $\langle x, y\rangle$ is the exploding vector field so is (II). The saddle like flows (III) are from $\langle y, x\rangle$. And finally the $\langle x-y, x-y\rangle$ all point in the same $\langle 1, 1\rangle$ direction which matchs (IV).

7. F #9 F03 Euler. Consider the vector field $\vec{F} = \langle -y - x/10, x - y/10\rangle$

   (a) Show $\vec{r}(t) = \left\langle e^{-t/10}\cos t, e^{-t/10}\sin t\right\rangle$ is a flow for $\vec{F}$

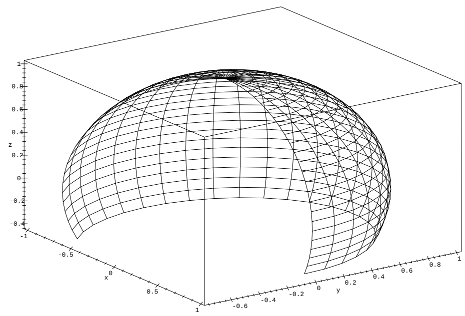
   (b) Use Euler's method to approximate the flow which starts at $(1, 0)$ by completing a table that starts like the one below with as much accuracy has your TI-89 can give. [Check to see that you are in both radian mode and using the Euler method]. Do five steps of size $\Delta t = 0.1$

This is the extra Euler problem from the homework, see the solution page for the extra problems (also on line). Here is what the TI-89 producted. Note five steps means six data lines.

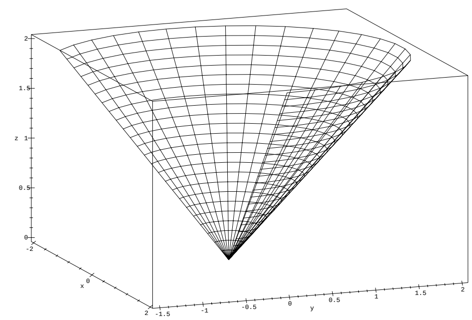
t	x	y
0.0	1.0	0.0
0.1	0.99	0.1
0.2	0.9701	0.198
0.3	0.9406	0.29303
0.4	0.90189	0.38416
0.5	0.85446	0.47051

8. T3#10 F98 Match the parameteric equations $\vec{r}(s,t) = \langle \cos t, \sin t, s\rangle$, $\vec{r}(s,t) = \langle s\cos t, s\sin t, s\rangle$, $\vec{r}(s,t) = \langle (2+s)\cos 2t, (2+s)\sin 2t, t\rangle$ and $\vec{r}(s,t) = \langle \sin s\cos t, \sin s\sin t, \cos s\rangle$ with the Maple plot3d's below. The range is always $s = 0..2, t = 0..4$.

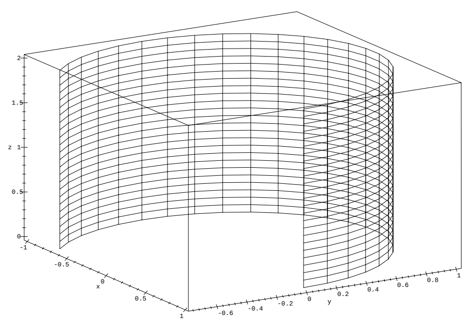
surface1



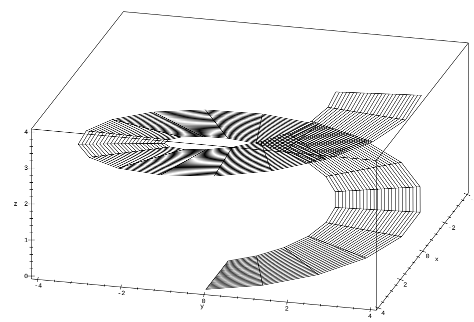
surface2



surface3



surface4



The $\langle \cos t, \sin t, s\rangle$ satisfies $x^2 + y^2 = 1$ so this is the cylinder – surface 3. The $\langle s\cos t, s\sin t, s\rangle$ satisfies $x^2 + y^2 = z^2$ so this is the cone – surface 2. The $\langle \sin s\cos t, \sin s\sin t, \cos s\rangle$, is a lot like spherical coordinates and indeed $x^2 + y^2 + z^2 = 1$ so this is the sphere – surface 1. Which of course is enough to complete this problem. But, if you let $s = 0$ in $\langle (2+s)\cos 2t, (2+s)\sin 2t, t\rangle$ you get $\langle 2\cos 2t, 2\sin 2t, t\rangle$ which is our old friend the helix. A helix could also be on surface 3, but not with the given grid pattern.