Practice Mini-Test 6 – Calculus 3 – Spring 04

1. T3#1 F02 Find curl $\vec{F}$ and find div $\vec{G}$, if the vector field $\vec{F} = \langle xy^2z^3, 0, x^3y^2z \rangle$ and if the vector field $\vec{G} = \langle z\sin(x/y), z\sin(x/y), z\sin(x/y) \rangle$. [Compare T3#1 F03 T3#1 S02 T3#1 F02]

2. T3#2 F03 Vector, scalar or nonsense. $f = f(x,y,z)$ and $g = g(x,y,z)$ are scalar fields and $\vec{F} = \vec{F}(x,y,z)$ and $\vec{G} = \vec{G}(x,y,z)$ are vector fields. Determine if the given object is a scalar field, a vector field or nonsense. [Compare T3#2 S02,T3#3 F02]

A. $f+g$	B. $\vec{F}+\vec{G}$	C. $f+\vec{G}$	D. $\nabla\vec{F}$	E. div g
F. curl(curl $\vec{G}$)	G. curl(div $\vec{G}$)	H. div(curl $\vec{F}$)	I. grad(div $\vec{F}$)	J. div(grad g)

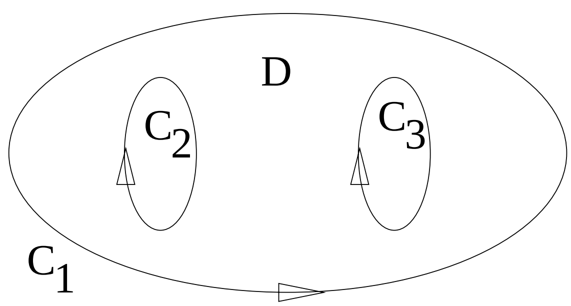
3. T3#3 F03 Find a function f so that $\nabla f = \vec{F}$ and use it to compute the line intergal $\int_C \vec{F}\cdot d\vec{r}$ for the vector field $\vec{F} = \langle y, x+6y+z^2, 2yz+1/z \rangle$ where C is some very complex counterclockwise corkscrew curve going from $(1,3,2)$ to $(5,-1,3)$. [Compare T3#2 F02]

4. T3#5 F03 Compute the line intergal $\int_C \vec{F}\cdot d\vec{r}$ for the vector field $\vec{F} = \langle y, z, x \rangle$ and C is the is the line from $(1,3,2)$ to $(5,-1,3)$. [Compare T3#3 S02]

5. T3#6 F03 Compute the flux of the vector field $\vec{F} = y\hat{\imath} + (1+z)\hat{k}$ through the part of the plane $3x+3y+z=3$ oriented outward with $x \geq 0, y \geq 0, z \geq 0$ [Compare T3#10 F02]

6. T3#7 F03 Use your TI-89 to graph the closed curve C given by $\vec{r}(t) = \langle \cos^3 t, \sin^3 t \rangle$ for $0 \leq t \leq 2\pi$. For the vector field $\vec{F} = \langle 0, x \rangle$ write and simplify the line integeral $\oint_C \vec{F}\cdot d\vec{r}$ down to a regular integral in t. Use your TI-89 to evaluate this integral. Finally use Green's theorem to explain why your answer is the area of the region enclosed by C. [Compare T3#10 S03 T3#9 F02 T3#9 S02]

7. T3#8 F03 The domain D of area 13 is the region inside C_1 but outside C_2 and C_3 (see below left) and $\vec{F} = \langle 7x^2 - 5y, 3x + 11y^2 \rangle$, find the line integral $\oint_{C_1} \vec{F}\cdot d\vec{r}$ if $\oint_{C_2} \vec{F}\cdot d\vec{r} = -15$ and $\oint_{C_3} \vec{F}\cdot d\vec{r} = -25$.



8. T3#5 S03 The vector field $\vec{F} = \langle 5, 2, 3 \rangle$ compute the flux of $\vec{F}$ over each of the surfaces S described below without evaluation any integrals. When there is a choice of normal, pick the normal whose dot product with $\vec{F}$ is positive.

   (a) S is the rectangle $\{(0,y,z), 1 \leq y \leq 3, 0.5 \leq z \leq 1\}$.

   (b) S is a circle in the xz-plane with radius 5.

   (c) S is a pentagon of area P with normal in the direction of $\langle 5, -5, -5 \rangle$

   [Compare T3#5 F02 T3#5 S02]

9. T3#6 S03 Use a theorem to show that if V is a 3D region in space with outward oriented boundry given by S then

$$\iint_S \langle x, y, z \rangle \cdot d\vec{A} = 3 \text{ Volume}(V)$$

   [Compare T3#7 F02]

10. T3#7 S03 Read this problem carefully. You are to sketch both of the vector fields and both the curves and explain the answers below but **NOT** by computing the line integrals. The two vector fields, let $\vec{F} = \langle x, 0\rangle$ and let $\vec{G} = \langle 0, x\rangle$. The two curves, let C_1 be the straight line from $(1,1)$ to $(3,1)$ and let C_2 be the circle or radius $2/\sqrt{\pi}$ centered at $(1,1)$ oriented counterclockwise. The following integrals were all be computed without integrating a single line integral, either by the fundemental theorem of calculus for line integrals, or by geometry, or by Green's theorem. Explain how each integral below was obtained without doing a single line integral. [Hint: start by checking if $\vec{F}$ or $\vec{G}$ is a gradient vector field.]

$$(A)\int_{C_1} \vec{F}\cdot\ d\vec{r} = 4 \qquad (B)\int_{C_1} \vec{G}\cdot\ d\vec{r} = 0 \qquad (C)\oint_{C_2} \vec{F}\cdot\ d\vec{r} = 0 \qquad (D)\oint_{C_2} \vec{G}\cdot\ d\vec{r} = 4$$

11. T3#10 F03 Divergence Theorem. Let V be volume inside the cylinder $x^2+y^2=4$ between $z=0$ and $z=3$. The boundry of V is oriented with outward pointing normal and divided into pieces as follows. Let B be the surface on the bottom of the cylinder, T be the surface at the top of the cylinder and S be the surface at the side of the cylinder. (See graphs below.) The vector field is $\vec{F} = \left\langle xy^2, xz^2, x^2z\right\rangle$ compute the following three integrals, the first without integrating, the last two by converting to polar or cylindrical coordinates.

$$\iint_B \vec{F}\cdot d\vec{A} \qquad \iint_T \vec{F}\cdot d\vec{A} \qquad \iiint_V \text{div } \vec{F}\ dV$$

Using the Divergence Theorem you can use the information above to find the value of the flux over S. Do so, and also write the and simplify the flux integral over S. Do **NOT** evaluate this last integral. [Well, using the TI-89 to evaluate the integral would be a way of checking your answer.]