1. T3#1 F02 Find curl \( \vec{F} \) and find div \( \vec{G} \), if the vector field \( \vec{F} = \langle xy^2z^3, 0, x^3y^2z \rangle \) and if the vector field \( \vec{G} = \langle z \sin(x/y), z \sin(x/y), z \sin(x/y) \rangle \). [Compare T3#1 F03 T3#1 S02 T3#1 F02]

2. T3#2 F03 Vector, scalar or nonsense. \( f = f(x, y, z) \) and \( g = g(x, y, z) \) are scalar fields and \( \vec{F} = \vec{F}(x, y, z) \) and \( \vec{G} = \vec{G}(x, y, z) \) are vector fields. Determine if the given object is a scalar field, a vector field or nonsense. [Compare T3#2 S02, T3#3 F02]

A. \( f + g \)  
B. \( \vec{F} + \vec{G} \)  
C. \( f + \vec{G} \)  
D. \( \nabla \cdot \vec{F} \)  
E. \( \text{div} \ g \)  
F. \( \text{curl} (\text{curl} \ \vec{G}) \)  
G. \( \text{curl} (\text{div} \ \vec{G}) \)  
H. \( \text{div} (\text{curl} \ \vec{F}) \)  
I. \( \text{grad} (\text{div} \ \vec{F}) \)  
J. \( \text{div} (\text{grad} \ g) \)

3. T3#3 F03 Find a function \( f \) so that \( \nabla f = \vec{F} \) and use it to compute the line integral \( \int_C \vec{F} \cdot d\vec{r} \) for the vector field \( \vec{F} = \langle y, x + 6y + z^2, 2yz + 1/z \rangle \) where \( C \) is some very complex counterclockwise corkscrew curve going from \( (1, 3, 2) \) to \( (5, -1, 3) \). [Compare T3#2 F02]

4. T3#5 F03 Compute the line integral \( \int_C \vec{F} \cdot d\vec{r} \) for the vector field \( \vec{F} = \langle 0, x \rangle \) and \( C \) is the line from \( (1, 3, 2) \) to \( (5, -1, 3) \). [Compare T3#3 S02]

5. T3#6 F03 Compute the flux of the vector field \( \vec{F} = y\hat{i} + (1 + z)\hat{k} \) through the part of the plane \( 3x + 3y + z = 3 \) oriented outward with \( x \geq 0, y \geq 0, z \geq 0 \) [Compare T3#10 F02]

6. T3#7 F03 Use your TI-89 to graph the closed curve \( C \) given by \( \vec{r}(t) = \langle \cos^3 t, \sin^3 t \rangle \) for \( 0 \leq t \leq 2\pi \). For the vector field \( \vec{F} = \langle 0, x \rangle \) write and simplify the line integral \( \oint_C \vec{F} \cdot d\vec{r} \) down to a regular integral in \( t \). Use your TI-89 to evaluate this integral. Finally use Green’s theorem to explain why your answer is the area of the region enclosed by \( C \). [Compare T3#10 S03 T3#9 F02 T3#9 S02]

7. T3#8 F03 The domain \( D \) of area 13 is the region inside \( C_1 \) but outside \( C_2 \) and \( C_3 \) (see below left) and \( \vec{F} = \langle 7x^2 - 5y, 3x + 11y^2 \rangle \), find the line integral \( \oint_{C_1} \vec{F} \cdot d\vec{r} \) if \( \oint_{C_2} \vec{F} \cdot d\vec{r} = -15 \) and \( \oint_{C_3} \vec{F} \cdot d\vec{r} = -25 \).

8. T3#5 S03 The vector field \( \vec{F} = \langle 5, 2, 3 \rangle \) compute the flux of \( \vec{F} \) over each of the surfaces \( S \) described below without evaluation any integrals. When there is a choice of normal, pick the normal whose dot product with \( \vec{F} \) is positive.

   (a) \( S \) is the rectangle \( \{(0, y, z), 1 \leq y \leq 3, 0.5 \leq z \leq 1\} \).
   (b) \( S \) is a circle in the \( xz \)-plane with radius 5.
   (c) \( S \) is a pentagon of area \( P \) with normal in the direction of \( \langle 5, -5, -5 \rangle \)

   [Compare T3#5 F02 T3#5 S02]

9. T3#6 S03 Use a theorem to show that if \( V \) is a 3D region in space with outward oriented boundary given by \( S \) then

   \[ \oiint \langle x, y, z \rangle \cdot d\vec{A} = 3 \text{ Volume } (V) \]

   [Compare T3#7 F02]
10. T3#7 S03 Read this problem carefully. You are to sketch both of the vector fields and both the curves and explain the answers below but **NOT** by computing the line integrals. The two vector fields, let \( \vec{F} = (x, 0) \) and let \( \vec{G} = (0, x) \). The two curves, let \( C_1 \) be the straight line from \((1,1)\) to \((3,1)\) and let \( C_2 \) be the circle or radius \( 2/\sqrt{\pi} \) centered at \((1,1)\) oriented counterclockwise. The following integrals were all computed without integrating a single line integral, either by the fundamental theorem of calculus for line integrals, or by geometry, or by Green’s theorem. Explain how each integral below was obtained without doing a single line integral. [Hint: start by checking if \( \vec{F} \) or \( \vec{G} \) is a gradient vector field.]

(A) \( \int_{C_1} \vec{F} \cdot d\vec{r} = 4 \)  
(B) \( \int_{C_1} \vec{G} \cdot d\vec{r} = 0 \)  
(C) \( \oint_{C_2} \vec{F} \cdot d\vec{r} = 0 \)  
(D) \( \oint_{C_2} \vec{G} \cdot d\vec{r} = 4 \)

11. T3#10 F03 Divergence Theorem. Let \( V \) be volume inside the cylinder \( x^2 + y^2 = 4 \) between \( z = 0 \) and \( z = 3 \). The boundary of \( V \) is oriented with outward pointing normal and divided into pieces as follows. Let \( B \) be the surface on the bottom of the cylinder, \( T \) be the surface at the top of the cylinder and \( S \) be the surface at the side of the cylinder. (See graphs below.) The vector field is \( \vec{F} = (xy^2, xz^2, x^2z) \) compute the following three integrals, the first without integrating, the last two by converting to polar or cylindrical coordinates.

\[
\int\int_{B} \vec{F} \cdot d\vec{A} \quad \int\int_{T} \vec{F} \cdot d\vec{A} \quad \int\int\int_{V} \text{div} \vec{F} \, dV
\]

Using the Divergence Theorem you can use the information above to find the value of the flux over \( S \). Do so, and also write the and simplify the flux integral over \( S \). Do **NOT** evaluate this last integral. [Well, using the TI-89 to evaluate the integral would be a way of checking your answer.]