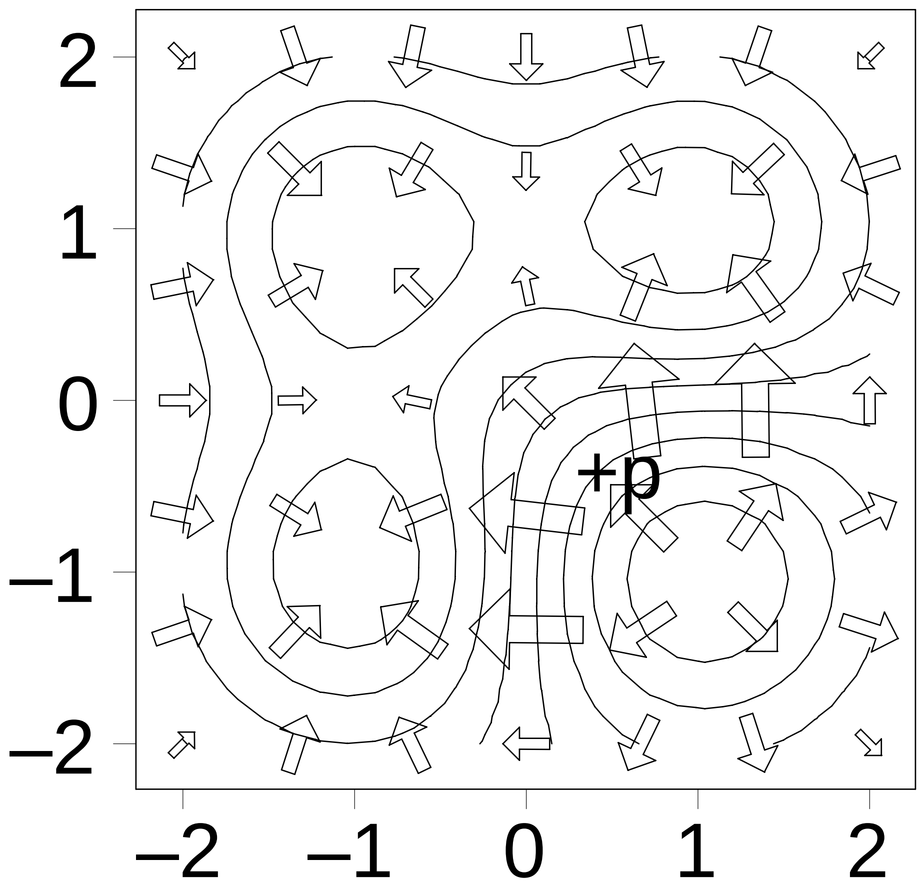
MAC 2313 Calculus 3 **Test 3** 23 Feb 2004

**Directions:** Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Find the directional derivative of $f(x, y, z) = x^2y^3 + z\ln(z + y)$ as one leaves the point $P = (1, -2, 3)$ heading in the direction of the point $Q = (0, -4, 2)$. An exact simplified answer please.
2. Use the chain rule (as shown in class) to find $\partial q/\partial u$ and $\partial q/\partial v$ if $q = r/s + st^2$, $r = e^{-u}$, $s = e^v \sin u$ and $t = e^u \cos v$
3. The graph below is combines a contour graph and a plot of the gradient of the function $f(x, y)$.
   (a) Find and classify all the local extrema (saddles, local minumins and local maximums) of $f(x, y)$
   (b) The point $p(1/2, -1/2)$ is shown on the graph, determine the sign (positive, negative or zero) of these partial or directional derivatives of $f$ at $p$: $f_x$, $f_y$, $f_{xx}$, $f_{yy}$, $f_{\langle 1/\sqrt{2}, 1/\sqrt{2}\rangle}$ and $f_{\langle -1/\sqrt{2}, 1/\sqrt{2}\rangle}$.




Problems #4 & #5 are on the other side of the test

4. True or False and a brief reason why or why not.

   (a) The function $f(x,y) = x^6 + y^2$ has a (global) maximum value for $(x,y) \in \mathbb{R}^2$
   (b) The function $f(x,y) = -x^6 - y^2$ has a (global) maximum value for $(x,y) \in \mathbb{R}^2$
   (c) The boundary $\partial D$ of the disk $D = \{(x,y)|x^2 + y^2 < 16\}$ is the circle $\{(x,y)|x^2 + y^2 = 4\}$.
   (d) The boundary $\partial D$ of the filled square $D = \{(x,y)|0 \leq x < 1, 0 \leq y < 1\}$ is the union of the line seqments $\{(1,y)|0 \leq y \leq 1\} \cup \{(x,1)|0 \leq x \leq 1\}$.
   (e) If the partial derivatives of $f$ are given by $f_x = (x-1)(x+y)(y^2-1)$ and $f_y = (x+1)(x+y-1)(y-1)$ then $(1,-1)$ would be a critical point of $f(x,y)$.
   (f) If $(a,b)$ is a critical point of $f$ and $f_{xx}(a,b) = 0$ but $D \neq 0$ then $(a,b)$ must a saddle point for $f$.
   (g) Let $g(x,y) = x^2 + y^2 = 2$, then $D = \{(x,y)|g(x,y) = 2\}$ is closed and bounded and suppose $f$, $f_x$ and $f_y$ are continuous on $D$ and $f$ is non-constant on $D$. Using Lagrange multipliers optimizing $f(x,y)$ given $x^2 + y^2 = 2$, will always give at least two critical points on $D$, that is points where $\nabla f$ is parallel to $\nabla g$.
   (h) If $f(x,y)$ has a (global) minimum value of 5 on a closed bounded set $D$ and this occurs at the point $(3,4)$ (so $f(3,4) = 5$), then $f(x,y)$ must have a local minimum at $(3,4)$.
   (i) Let $P$ be a point in $\mathbb{R}^3$, let the temperature in $\mathbb{R}^3$ is given by $T(x,y,z)$ and let $\vec{u} = \nabla T(P)/\|\nabla T(P)\|$. If the temperature at $P$ is 86 and you want to decrease the the temperature as fast as possible, then you should move in the $\vec{u}$ direction.
   (j) The same as problem above, but this time you want to move in a direction which will keep the temperature a constant 86 degrees, then you can move in any direction perpendicular to the $-\vec{u}$ direction.

5. Find the critical points of the function $f(x,y) = x^2y - x^2 + y - y^2$ and classify these local extrema by filling out a table like the one below using a separate row for each critical point.

| Points | $f_{xx}$ | $f_{yy}$ | $f_{xy}$ | $D$ | classification |
|---|---|---|---|---|---|
| $(?,?)$ | ? | ? | ? | ? | ? |