1. Find the directional derivative of \( f(x, y, z) = x^2y^3 + z\ln(z + y) \) as one leaves the point \( P = (1, -2, 3) \) heading in the direction of the point \( Q = (0, -4, 2) \). An exact simplified answer please.

2. Use the chain rule (as shown in class) to find \( \partial q/\partial u \) and \( \partial q/\partial v \) if \( q = r/s + st^2 \), \( r = e^{-u} \), \( s = e^v \sin u \) and \( t = e^u \cos v \).

3. The graph below is combines a contour graph and a plot of the gradient of the function \( f(x, y) \).

   (a) Find and classify all the local extrema (saddles, local minumins and local maximums) of \( f(x, y) \).
   (b) The point \( p(1/2, -1/2) \) is shown on the graph, determine the sign (positive, negative or zero) of these partial or directional derivatives of \( f \) at \( p \): \( f_x, f_y, f_{xx}, f_{yy}, f_{1/\sqrt{2}, 1/\sqrt{2}} \) and \( f_{-1/\sqrt{2}, 1/\sqrt{2}} \).
4. True or False and a brief reason why or why not.

(a) The function \( f(x, y) = x^6 + y^2 \) has a (global) maximum value for \((x, y) \in \mathbb{R}^2 \).

(b) The function \( f(x, y) = -x^6 - y^2 \) has a (global) maximum value for \((x, y) \in \mathbb{R}^2 \).

(c) The boundary \( \partial D \) of the disk \( D = \{(x, y)|x^2 + y^2 < 16\} \) is the circle \( \{ (x, y)|x^2 + y^2 = 4 \} \).

(d) The boundary \( \partial D \) of the filled square \( D = \{(x, y)|0 \leq x < 1, 0 \leq y < 1\} \) is the union of the line segments \( \{(1, y)|0 \leq y \leq 1\} \cup \{(x, 1)|0 \leq x \leq 1\} \).

(e) If the partial derivatives of \( f \) are given by \( f_x = (x-1)(x+y)(y^2-1) \) and \( f_y = (x+1)(x+y-1)(y-1) \) then \((1, -1)\) would be a critical point of \( f(x, y) \).

(f) If \((a, b)\) is a critical point of \( f \) and \( f_{xx}(a, b) = 0 \) but \( D \neq 0 \) then \((a, b)\) must a saddle point for \( f \).

(g) Let \( g(x, y) = x^2 + y^2 = 2 \), then \( D = \{(x, y)|g(x, y) = 2\} \) is closed and bounded and suppose \( f, f_x \) and \( f_y \) are continuous on \( D \) and \( f \) is non-constant on \( D \). Using Lagrange multipliers optimizing \( f(x, y) \) given \( x^2 + y^2 = 2 \), will always give at least two critical points on \( D \), that is points where \( \nabla f \) is parallel to \( \nabla g \).

(h) If \( f(x, y) \) has a (global) minimum value of 5 on a closed bounded set \( D \) and this occurs at the point \((3, 4)\) (so \( f(3, 4) = 5 \)), then \( f(x, y) \) must have a local minimum at \((3, 4)\).

(i) Let \( P \) be a point in \( \mathbb{R}^3 \), let the temperature in \( \mathbb{R}^3 \) is given by \( T(x, y, z) \) and let \( \vec{u} = \frac{\nabla T(P)}{\|\nabla T(P)\|} \).

If the temperature at \( P \) is 86 and you want to decrease the the temperature as fast as possible, then you should move in the \( \vec{u} \) direction.

(j) The same as problem above, but this time you want to move in a direction which will keep the temperature a constant 86 degrees, then you can move in any direction perpendicular to the \( -\vec{u} \) direction.

5. Find the critical points of the function \( f(x, y) = x^2y - x^2 + y - y^2 \) and classify these local extrema by filling out a table like the one below using a separate row for each critical point.

<table>
<thead>
<tr>
<th>Points</th>
<th>( f_{xx} )</th>
<th>( f_{yy} )</th>
<th>( f_{xy} )</th>
<th>( D )</th>
<th>classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>(?, ?)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
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