MAC 2313 Calculus 3 **Test 4** 16 Mar 2005

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

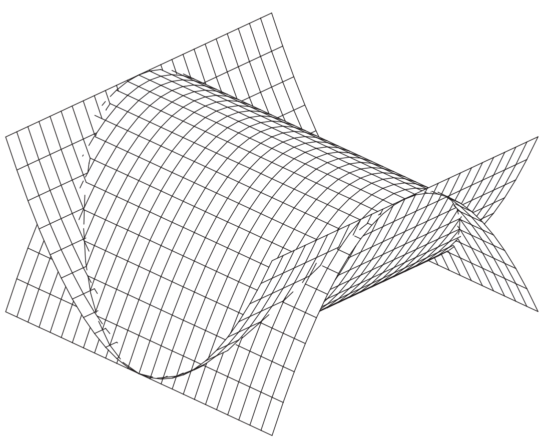
1. Sketch the region of integration for the integral below, then change the order of integration (do NOT evaluate) and then convert the double integral to polar cordinates (do NOT evaluate).

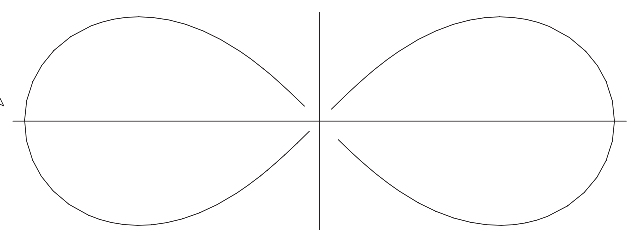
$$\int_0^3 \int_{-x}^{x} x^2 + y^2 \, dy \, dx$$

2. Let $D = \{(x,y) | x^2 + y^2 \leq 1\}$ be the unit disk, $B = \{(x,y) \in D | y \leq 0\}$ be the bottom half of D, $R = \{(x,y) \in D | x \geq 0\}$ be the right half of D, and $Q = \{(x,y) \in D | y \leq 0, x \geq 0\}$ the part of D in the fourth quadrant. Decide if the following double integrals are positive, negative or zero.

$A = \iint_D y \, dA$ $B = \iint_B x^3 \, dA$ $C = \iint_Q xy \, dA$ $D = \iint_B \sin x \, dA$ $E = \iint_D 1 - x^2 - y^2 \, dA$
$F = \iint_B -y \, dA$ $G = \iint_R y^2 \, dA$ $H = \iint_R -5x \, dA$ $I = \iint_R \sin x \, dA$ $J = \iint_D x - 0.005 \, dA$

3. Write and evaluate a double integral which will give the volume under the parabolic cylinder $z = 1 - x^2$ and above the parabolic cylinder $z = y^2$. (See the graph below left)





4. Setup and evaluate a double integral which will compute the area of one loop of the leminscate $r^2 = 2\cos 2\theta$ (see the graph above right).

5. Find the center of mass of a lamina with constant density $\delta(x,y) = k$ that occupies the region under the curve $y = \cos x$ from $x = -\pi/2$ to $x = \pi/2$.