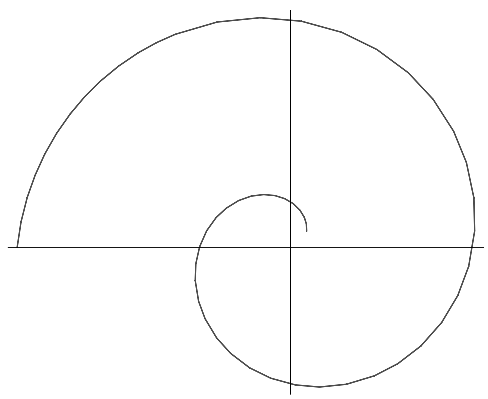
MAC 2313 Calculus 3 **Test 6** 13 Apr 2005

**Directions:** Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Complete the following steps to compute the flux integral below. (Hint: no actual integration is required below.)
$$\iint_S (\operatorname{curl} \vec{F}) \cdot d\vec{S}$$
   (a) Compute $\operatorname{curl} \vec{F}$ for $\vec{F} = \langle -y, x, z \rangle$.
   (b) Compute the area of $S$ if the surface $S$ is the equilateral triangle with vertices $P(1,0,0)$, $Q(0,1,0)$ and $R(0,0,1)$.
   (c) Compute the upward unit normal $\vec{n}$ for surface $S$ above.
   (d) Show $\operatorname{curl} \vec{F} \cdot \vec{n}$ is a constant on the surface $S$.
   (e) Finally compute the flux using two of the numbers above.

2. Find $f$ so that $\nabla f = \vec{F} = \langle y, x+y \rangle$ and use $f$ to find the line integral $\int_C \vec{F} \cdot d\vec{r}$ if $C$ is the curve pictured below from $(2,2)$ to $(-20,0)$.



3. Vector, scalar or nonsense. $P$ and $Q$ are points in 3-space, $f = f(x,y,z)$ is a scalar fields, $\vec{r}(t)$ or $\vec{r}(u,v)$ is a parametric equation of a curve $C$ or a surface $S$, and $\vec{F} = \vec{F}(x,y,z)$ and $\vec{G} = \vec{G}(x,y,z)$ are vector fields. Determine if the given object is a scalar field, a vector field or nonsense.

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|---|---|---|---|---|---|---|---|---|---|
| A. | $\frac{\partial f}{\partial x}$ | B. | $\vec{F} \times \vec{G}$ | C. | $\vec{F}\vec{G}$ | D. | $\frac{\partial \vec{r}}{\partial u}$ | E. | $\vec{F} - \vec{G}$ |
| F. | $\operatorname{curl} f$ | G. | $\operatorname{grad} \vec{G}$ | H. | $\vec{F} \cdot \vec{G}$ | I. | $\operatorname{div} \operatorname{grad}(f)$ | J. | $\operatorname{curl} \operatorname{curl} \vec{F}$ |

4. By evaluating both integrals, check Green's theorem
$$\oint_{\partial D} \vec{F} \cdot d\vec{r} = \iint_D Q_x - P_y \, dA$$
   when $D = \{(x,y) : x^2 + y^2 \le 16\}$ is the disk of radius 4 and $\vec{F} = \langle -xy, 2x \rangle$.

5. Find the flux of $\vec{F} = \langle 2, y, 2z \rangle$ over the upward oriented surface $S$ given by the portion of $z = 1 - x^2 - y^2$ that lies in the first octant.

   Explicitly give $\vec{r}$, your parametrization with limits, your $d\vec{S}$ with the correct sign for the normal orientation and with the correct differentials. and your $\vec{F}(\vec{r})$.