MAS 4106 Linear Algebra 2 **Quiz 3** 15 Feb 2006 **Name:** ____________________

Show **ALL** work for credit; Give exact answers when possible.

1. The matrix A has eigenvalues $\lambda_1 = 5$ and $\lambda_2 = -3$ with eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and let $\vec{w} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

   (a) Solve the differenece equation, the discrete dynamic system $x_{k+1} = Ax_k$ with initial condition $x_0 = \vec{w}$.

   (b) Solve the matrix differential equation $x'(t) = Ax(t)$ with initial condition $x(0) = \vec{w}$.

2. True or False. The 3×5 matrix $B = [\vec{v}_1, \vec{v}_2, \ldots \vec{v}_5]$ row reduces to $\begin{bmatrix} 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Let A be an $n \times n$ matrix with real entries.

   (a) If $(A - \lambda I)^3 \vec{v} = 0$ then $(A - \lambda I)^4 \vec{v} = 0$.

   (b) If $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n = 0$ is true for some choice of scalars $c_1, c_2, \ldots c_n$ then $\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n\}$ is linearly dependent.

   (c) If $\{\vec{v}, \vec{w}\}$, $\{\vec{u}, \vec{v}\}$ and $\{\vec{u}, \vec{w}\}$ are all linearly independent, then $\{\vec{u}, \vec{v}, \vec{w}\}$ is also linearly independent.

   (d) A has at least one eigenvector.

   (e) $\mathbb{R}^n$ has a basis of eigenvectors of A.

   (f) A basis for the kernel of B is $\{[1, 0, 0, 0, 0]^T, [0, -2, 0, 1, 0]^T, [0, -3, 3, 0, 1]^T\}$.

   (g) A basis for the column space of B is $\{[1, 0, 0]^T, [0, 1, 0]^T\}$.

   (h) For every λ there is an integer $k \le n$ so that $\ker(A - \lambda I)^k = \ker(A - \lambda I)^{k+1}$

   (i) If $\vec{v}$ is a vector so that $(A - 5I)\vec{v} \ne 0$ but $(A - 5I)^2\vec{v} = 0$ then $\vec{v}$ is a generalized eigenvector of A order 2 for the eigenvalue 5.

   (j) The matrix differential equation $x' = Ax$ has the origin as an attractor if all the eigenvalues λ of A satisfy $|\lambda| < 1$.