MAS 4106 Linear Algebra 2 **Quiz 4** 17 Mar 2006 **Name:** _____
Show **ALL** work for credit; Give exact answers when possible.

1. True or False.
   (a) The matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is positive definite.
   (b) If A is any $n \times m$ matrix, then $A^T A$ is positive semi-definite.
   (c) Every quadratic form can be written $x^T Ax$ for some symmetric matrix A.
   (d) Orthogonal matrices actually have orthonormal columns.
   (e) For a positive definite matrix, the eigenvalues and singular values are never the same.
   (f) Only symmetric matrices are orthogonally diagonizable.
   (g) If P is an $n \times n$ matrix and $P^{-1} = P^T$, then P is an orthogonal matrix.
   (h) Each orthogonal matrix is symmetric.
   (i) The maximum value of the quadratic form $Q(x) = x^T Ax$, (A symmetric), constrained to the n-sphere $x^T x = 1$ is the largest eigenvalue in absolute value of A.
   (j) If A has eigenvalues 9 and -4, then A has singular values 3 and 2.

2. Find the singular value decomposition for the matrix A, that is find the orthogonal matrices U and V and a diagonal-like matrix Σ so that $A = U\Sigma V^T$, when

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$