MAS 4106 Linear Algebra 2 **Test 1** 24 Feb 2006

Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Find the stochastic matrix and steady state vector.
   Students live either on-campus or off campus. Every year 40% of the on-campus students move off-campus (and the rest stay on-campus) and 30% of the off-campus students move on-campus (and the rest stay off-campus).

2. Solve the system of differential equations $x'(t) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} x(t)$ with initial condition $x(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

3. True or False.

   (a) If $a, b \in \mathbb{R}$, so $z = a - bi \in \mathbb{C}$, then $e^z = e^a(\cos b - i \sin b)$.

   (b) The set of continuous functions f with $\int_a^b f(t)\,dt = 1$ is a subspace of $C[a,b]$.

   (c) If f is an odd function, then all of the coefficients of the $\sin nt$ terms in the Fourier series approximation to f on $C[-\pi,\pi]$ will be 0.

   (d) Either $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly independent or there are constants c_2, c_3 and c_4, not all zero, so that $\vec{v}_1 = c_2\vec{v}_2 + c_3\vec{v}_3 + c_4\vec{v}_4$

   (e) Let $P = \{f \in C[a,b] : \int_a^b f(t)\,dt \geq 0\}$, if f and g are in P, then $f + g \in P$

   (f) The function te^t is a generalized eigenvector of order 2 for the eigenvalue 1 and the differentiation operator $Df = f'(t)$.

   (g) If $e_1, \dots e_n$ is an orthonormal basis for the subspace $S \subset V$ and $v \in V \backslash S$, then the vector in S closest to v is $\langle e_1, v\rangle\, e_1 + \cdots \langle e_n, v\rangle\, e_n$.

   (h) If $p(t) = t$ and $q(t) = t(1-t)$ and $\langle\cdot,\cdot\rangle$ is given by evaluation at $-3, -1, 1, 3$, then in this inner product $p(t) \perp q(t)$.

   (i) If A has 0 as a repeated eigenvalue with one independent eigenvector $\vec{v}_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\vec{v}_1 = A\begin{bmatrix} -1 \\ 5 \end{bmatrix}$ then $AP = PB$ where $P = \begin{bmatrix} 5 & 1 \\ -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

   (j) If A has $4 - 3i$ as an eigenvalue with eigenvector $\begin{bmatrix} 2+5i \\ 1 \end{bmatrix}$ then $AP = PC$ where $P = \begin{bmatrix} 2 & 5 \\ 1 & 0 \end{bmatrix}$ and $C = 5\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$.

4. Find the third-order Fourier approximation to the piecewise continuous function $f(t)$ (defined below) on $C[0, 2\pi]$.

$$f(t) = \begin{cases} 1, & 0 \leq t \leq \pi \\ 0, & \pi \leq t \leq 2\pi \end{cases}$$