MAC 3313 Calculus 3 **Test 1** 31 Jan 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. If the position function is $\mathbf{r}(t) = \left\langle t^3, t^2+1, t^3-1 \right\rangle$, find the velocity, the speed and the acceleration.

2. Find the equation of the plane through the point $(6, 5, -2)$ parallel to the plane $x + 2y - z + 1 = 0$.

3. Find the point where the line $x = 1 + t, y = 2t, z = 3t$ intersects the plane $3x - 2y + z = 9$.

4. Find the value of $x$ such that the vectors $\langle 2, x, 3\rangle$ and $\langle x, 8, 6\rangle$ are perpendicular and find the value $x$ such the vectors are parallel.

5. Find the scalar and vector projections of $\mathbf{b} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ onto $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}$.

6. Find parametric equations for the line of intersection of the planes $2x + 5z = -3$, and $x - 3y + z = -2$.

7. Identify and sketch the graph of the equation $x^2 + y^2 + z^2 = 2x$ and re-write the equation in both cylinderical and spherical co-ordinates.

8. Find the equation of the plane that passes through the point $(0, 1, 2)$ and contains the line $x = y - 1 = z$.

9. Find and simplify both the unit tangent vector $\mathbf{T}(t)$ and the curvature $\kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$ of the space curve $\mathbf{r}(t) = \left\langle t^2, 2t^3/3, t \right\rangle$.

10. Find and simplify the arclength of $\mathbf{r}(t) = \langle e^t, e^t sin(t), e^t cos(t)\rangle, 0 \leq t \leq 2\pi$.