MAC 3313 Calculus 3 **Test 3** 9 Apr 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Find the curl and div of $\mathbf{F} = \left\langle x^2y, yz^2, zx^2 \right\rangle$.

2. Find f so that $\mathbf{F} = \nabla f$ and use it to find the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$. Here $\mathbf{F} = \left\langle 2xz + \sin y, x\cos y, x^2 \right\rangle$ and C is a curve from $(1,0,0)$ to $(1,0,2\pi)$.

3. Evaluate the line integral $\int_C x^2y dx - 3y^2 dy$ using Green's Theorem when C is the curve which goes around the perimeter of the region $\{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$ in the backwards (clockwise) direction.

4. Find the equation of the tangent plane to the parametric surface given by $\left\langle u^2, u - v^2, v^2 \right\rangle$ at the point $(1,0,1)$.

5. Rewrite but do **NOT** evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ as an usual double iterated integral (including limits of integration and a simplified integrand). Here $\mathbf{F} = \langle y, x, xy \rangle$ and S is the portion of the paraboloid $z = x^2 + 2y^2$ over the region $\{(x,y) : 1 \le x \le 2, \ln x \le y \le \pi\}$ Use the upward pointing normal of S.

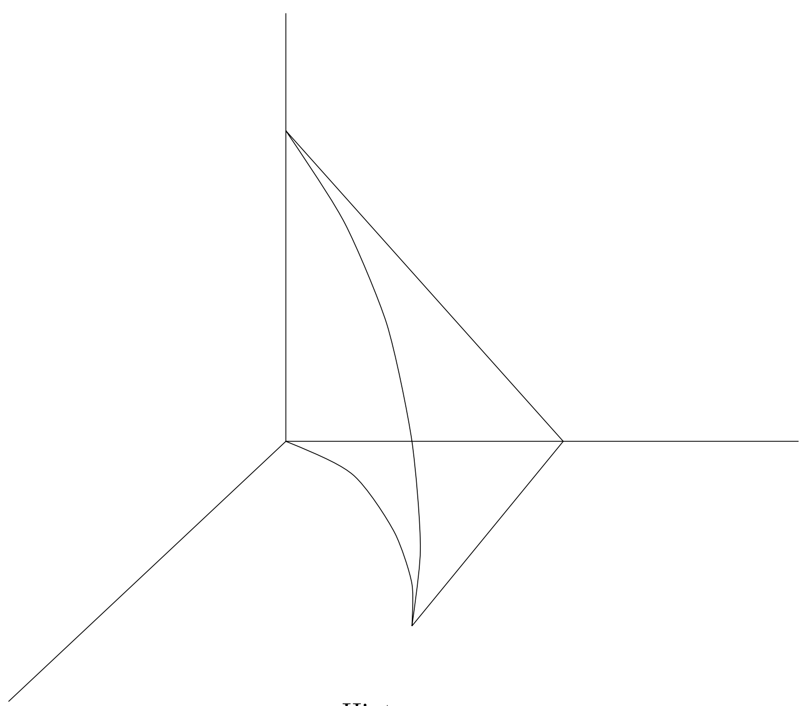
6. Set up but do **NOT** evaluate a double iterated integral for the surface area of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. The double iterated integral needs to have limits of integration and a simplified integrand.

7. Use cylindrical co-ordinates to evaluate $\int\int\int_E x^2 dV$ when E is the solid within $x^2 + y^2 = 1$, above $z = 0$ and below $z^2 = 4x^2 + 4y^2$.

8. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = \left\langle x^2y, -xy \right\rangle$, and $\mathbf{r}(t) = \left\langle t^3, t^4 \right\rangle, 0 \le t \le 1$.

9. Use the given transformation to evaluate $\int\int_R x dA$ where R is the region in the **FIRST** quadrant where $9x^2 + 4y^2 \le 36$ and the transformation is $x = 2u, y = 3v$. Also explicitly draw R and S, the region in the uv plane that maps to R in the xy plane by this transformation. Clearly label the Jacobian of the transformation.

10. Rewrite the the limits of $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z) dz dy dx$ in the orders $dxdydz$ and $dydzdx$.



Hint