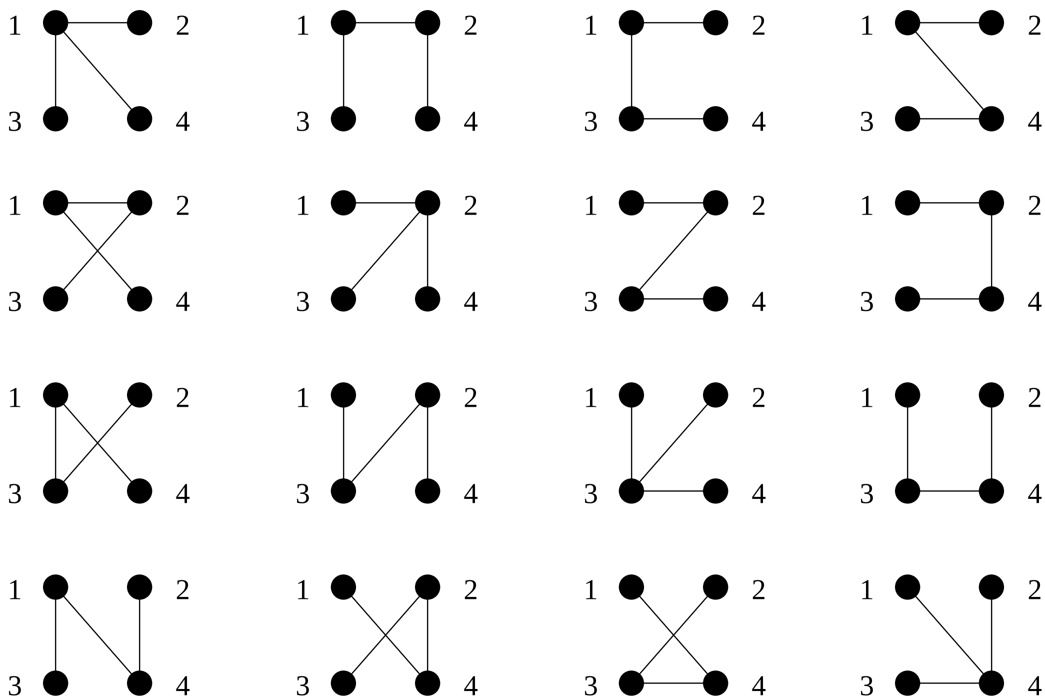
HW6

3.15: The spanning trees are listed below.



3.16: The spanning tree in row i and col j above has the Prüfer code ij.

3.22: For each edge e we divide the spanning trees of G into two subsets, those which contain e and those that do not. Clearly a spanning tree of G which does not contain e is also a spanning tree of $G - e$. Conversely, a spanning tree of $G - e$ is a spanning tree of G which does not contain e. This is a 1-1 correspondence between spanning trees of G not containing e and spanning trees of $G - e$.

Suppose T is a spanning tree of G which does contain $e = uv$. The graph $T \circ e$ is still connected and has one less edge than vertex so it is a tree and it spans $G \circ e$. The problem is going backwards from a spanning tree of $G \circ e$ to a spanning tree of G which uses e when $G \circ e$ is a multi-graph and not a graph. Let $\bar{e}$ be the new vertex of $G \circ e$ and label each edge incident to $\bar{e}$ with either u or v depending on if it came from an edge incident to u or v respectively.

Suppose S is a spanning tree of the multi-graph $G \circ e$. Construct T from S as follows. Any edge in S not incident to $\bar{e}$ is also an edge of T. The edges $w\bar{e}$ in S incident to $\bar{e}$ are labeled u or v which correspond to wu or wv respectively in T. Add the edge e to T, so T has the correct number of edges for a tree. Furthemore, this construction cannot create a cycle so T is a spanning tree of G. This is a 1-1 correspondence between spanning trees of G containing e and spanning trees of $G \circ e$.