3.15: The spanning trees are listed below.

3.16: The spanning tree in row \( i \) and col \( j \) above has the Prüfer code \( ij \).

3.22: For each edge \( e \) we divide the spanning trees of \( G \) into two subsets, those which contain \( e \) and those that do not. Clearly a spanning tree of \( G \) which does not contain \( e \) is also a spanning tree of \( G - e \). Conversely, a spanning tree of \( G - e \) is a spanning tree of \( G \) which does not contain \( e \). This is a 1-1 correspondence between spanning trees of \( G \) not containing \( e \) and spanning trees of \( G - e \).

Suppose \( T \) is a spanning tree of \( G \) which does contain \( e = uv \). The graph \( T \circ e \) is still connected and has one less edge than vertex so it is a tree and it spans \( G \circ e \). The problem is going backwards from a spanning tree of \( G \circ e \) to a spanning tree of \( G \) which uses \( e \) when \( G \circ e \) is a multi-graph and not a graph. Let \( \bar{e} \) be the new vertex of \( G \circ e \) and label each edge incident to \( \bar{e} \) with either \( u \) or \( v \) depending on if it came from an edge incident to \( u \) or \( v \) respectively.

Suppose \( S \) is a spanning tree of the multi-graph \( G \circ e \). Construct \( T \) from \( S \) as follows. Any edge in \( S \) not incident to \( \bar{e} \) is also an edge of \( T \). The edges \( u\bar{e} \) in \( S \) incident to \( \bar{e} \) are labeled \( u \) or \( v \) which correspond to \( wu \) or \( wv \) respectively in \( T \). Add the edge \( e \) to \( T \), so \( T \) has the correct number of edges for a tree. Furthermore, this construction cannot create a cycle so \( T \) is a spanning tree of \( G \). This is a 1-1 correspondence between spanning trees of \( G \) containing \( e \) and spanning trees of \( G \circ e \).