HW7

2.14: Since G is n-connected it must have at least $n+1$ vertices. Otherwise removing fewer then n vertices would trivialize G. Suppose H is $G+x$ and the n new edges xv_i for $i = 1 \ldots n$. H has at least $n+2$ vertices. Suppose removing k vertices of H disconnects H. If one of the components is just the vertex x, then we must have removed each of the n edges xv_i so $n \leq k$. Otherwise, at least two of the components must contain vertices of G and hence the removal of these k vertices (or $k-1$ if x was one of them) must also disconnect G so $n \leq k$. Therefore H is n-connected. (Actually $\kappa(H) = n$ even when $\kappa(G) > n$.)

2.15: Do the construction above. Since H is n-connected there are n-internally disjoint vx-paths and each of these must use a different v_i by Menger's Theorem. (Since x and v are non-adjacent and it takes the removal of at least n vertices to separate x and v in H.) Ignoring the last edge in each path produces internally disjoint vv_i paths in G.

2.16: If G has only even vertices and the edge $e = uv$ is a bridge, then each of the two components of $G - e$ will have one odd vertex (both u and v are now the only odd vertices) which is impossible by the handshaking lemma.