1. For the IVP \( y' = t^2 + y^2 = f(t, y) \) and \( y(0) = 1 \) numerically (by hand) compute the solution with a stepsize of \( \Delta t = h = 1/2 \).
   
   (a) By Euler’s method by completing a table like the one below. See the example in part 2. Euler’s method uses \( hy'(t) = h f(t, y(t)) \approx \Delta y \) and so \( y(t + h) \approx y + \delta y = y(t) + hy'(t) \).

   \[
   \begin{array}{c|c|c|c|c}
   t & y(t) & y'(t) & \Delta y & y(t + h) \\
   \hline
   0 & & & & \\
   1/2 & & & & \\
   1 & & & & \\
   3/2 & & & & \\
   \end{array}
   \]

   (b) By RK’s method by completing a table like the one below. RK also uses \( y(t + h) \approx y + \Delta y \) but \( \Delta y = (\text{avg } y') h \) where

   \[
   k_1 = f(t, y(t)) \quad k_2 = f(t+h/2, y(t)+k_1h/2) \quad k_3 = f(t+h/2, y(t)+k_2h/2) \quad k_4 = f(t+h), y(t)+k_3h
   \]

   \[
   \text{avg } y' = (k_1 + 2k_2 + 2k_3 + k_4)/6
   \]

   \[
   \begin{array}{c|c|c|c|c|c|c|c|c}
   t & y(t) & k_1 & k_2 & k_3 & k_4 & \text{avg } y' & \Delta y & y(t + h) \\
   \hline
   0 & & & & & & & & \\
   1/2 & & & & & & & & \\
   1 & & & & & & & & \\
   3/2 & & & & & & & & \\
   \end{array}
   \]

2. We do an example that is almost identical. Our IVP is \( y' = 1 + y^2 = f(t, y) \) and \( y(0) = 0 \). Observe, first \( y(t) = \tan t \) is a solution to this equation and second that this solution blows up as \( t \to \pi/2 \) from the left.

   (a) Euler’s method table. Euler’s method uses \( hy'(t) = h f(t, y(t)) \approx \Delta y \) and so \( y(t+h) \approx y(t) + hy'(t) \).

   \[
   \begin{array}{c|c|c|c|c|c|c|c|c}
   t & y(t) & y'(t) & \Delta y & y(t + h) \\
   \hline
   0 & 0 & 1 & 1/2 & 1/2 \\
   1/2 & 1/2 & 5/4 & 5/8 & 9/8 \\
   1 & 9/8 & 145/64 & 145/128 & 289/128 \\
   3/2 & 289/128 & 99905/16384 & 99905/32768 & 173889/32768 \\
   \end{array}
   \]

   (b) RK’s method table. RK uses

   \[
   k_1 = f(t, y(t)) \quad k_2 = f(t+h/2, y(t)+k_1h/2) \quad k_3 = f(t+h/2, y(t)+k_2h/2) \quad k_4 = f(t+h), y(t)+k_3h
   \]

   \[
   \text{avg } y' = (k_1 + 2k_2 + 2k_3 + k_4)/6
   \]

   \[
   \begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
   t & y(t) & k_1 & k_2 & k_3 & k_4 & \text{avg } y' & \Delta y & y(t + h) \\
   \hline
   0 & 0 & 1 & 1.062500000 & 1.070556641 & 1.218132667 & 1.080707658 & 0.5403538290 & 0.5403538290 \\
   0.5 & b & 1.291982260 & 1.745372176 & 1.953936782 & 1.634010036 & 1.720768369 & 0.8603841845 & 1.400738014 \\
   \end{array}
   \]

   \( b \) is used here for 0.5403538290 so the table will fit on the page.