MAP 2302 Diff-E-Qs   **Lab 3**   Summer 2005

Directions: Use only **ONE** side of each page, use ink and a staple.

Welcome to phase space, and rabbits chasing carrots.

This is the standard predator prey equations let $R(t)$ be the number of rabbits (in the unusual predator role) and $C(t)$ be the number of carrots (in an unusual nomadic role). This system is governed by the equations where α, β, λ and μ are postitive constants.

$$\frac{dC}{dt} = \alpha C - \lambda C R$$
$$\frac{dR}{dt} = -\beta R + \mu C R$$

There is an equilibrium solution $C = \beta/\mu$ and $R = \alpha/\lambda$ which can be found by setting the right hand side to zero and solving the two equations in two unknowns. We are interesting in non- equilibrium solutions. Our sample values for the parameters are $\alpha = 1$, $\lambda = 0.03$, $\beta = 0.4$ and $\mu = 0.01$. Time runs for at least $0 \le t \le 12$, plots should show at least $0 \le x \le 140$, and $0 \le y \le 80$. Use a stepsize of 0.2

Don't forget to explain how you got your numbers from your technology. Also remember clarity and presidentation.

1. Graph I Start with $C(0) = R(0) = 15$ and plot both $C(t)$ and $R(t)$ versus (the extended) time $0 \le t \le 36$ on the same graph. Measure the peak to peak time for each function and phase shift between the two graphs.

2. Graph II, the phase plane. Plot (several) $(C(t), R(t))$ as a parameteric equation, that is $x = C(t)$ and $y = R(t)$. Do this for several initial values (C_0, R_0), namely $(15, 15), (20, 20), (25, 25)$ and $(30, 30)$. (All on one graph.) What does the peak to peak time represent? What is the phase?

3. Changing parameters

   (a) Starting with the original parameters. How will the solution change if α is increased? Increase α by 50% and describe the change you see.

   (b) Starting with the original parameters. How will the solution change if λ is increased? Double λ and describe the change you see.

   (c) Starting with the original parameters. How will the solution change if β is increased? Double β and describe the change you see.

   (d) Starting with the original parameters. How will the solution change if μ is increased? Increase μ by 50% and describe the change you see.

4. The solutions look periodic, but there could be tight spiral solutions between the cycles. Or it could one slow spirial either going into or out of the equilibrium point. Check this by finding the following data. Find when a couple of the curves in question 2 cross the line segment $C = \beta/\mu$ $0 \le R \le \alpha/\lambda$. Follow these 2 curves through 3 crossings, (so time needs to be $0 \le t \le 36$ roughly). We end up with coordinates $(\beta/\mu, R_1)$ $(\beta/\mu, R_2)$ and $(\beta/\mu, R_3)$.

initial	R_1	R_2	R_3	max diff
$(15, 15)$	?	?	?	?
$(20, 20)$	?	?	?	?

If $a < c < b$ then the line from (a, A) to (b, B) intersects $x = c$ and the point $(c, ((b - c)A + (c - a)B)/(b - a)$