Directions: Use only **ONE** side of each page, use ink and a staple. This lab should resonate resonance with good vibrations.

This lab is about linear approximations to non-linear second order ODEs. On page 21 of the text, the equation of an undamped pendulum is derived. We consider the unforced damped equation

$$y'' + y' + \sin y = 0 \qquad (A)$$

where y is the angle θ the pendulum makes with the horizontal. Since the Taylor series of

$$\sin y \approx y - \frac{y^3}{6} + \frac{y^5}{120} + \ldots$$

we have a number of approimations. The most important is the linear approximation

$$y'' + y' + y = 0 \qquad (B)$$

We also consider the fifth order approximation

$$y'' + y' + y - \frac{y^3}{6} + \frac{y^5}{120} = 0 \qquad (C)$$

and briefly the third order approximation

$$y'' + y' + y - \frac{y^3}{6} = 0 \qquad (D)$$

to expose short comings that truncations can cause.

We consider the initial value problem $y(0) = 0$ and $y'(0) = v$ where v takes on the increasing sequence of values 1, 3 and 5.

Don't forget to explain how you got your numbers from your technology. Also remember clarity and presentation.

1. Consider the case $v = 1$, In this range the equations A, B and C closely agree. Complete the table below to show this clossness. The next equilibrium is the next time t when $y(t) = 0$, likely you will need to interpolate to find t.

equation	maximum (t,y(t))	next equilibrium	minimum (t,y(t))	next equilibrium
A	$(?,?)$	$t = ?$	$(?,?)$	$t = ?$
B	$(?,?)$	$t = ?$	$(?,?)$	$t = ?$
C	$(?,?)$	$t = ?$	$(?,?)$	$t = ?$

2. Next consider $v = 3$, in this range A and C agree well but differ somewhat from B. The next equilibrium is the next time t when $y(t) = 0$. Complete the same table

equation	maximum (t,y(t))	next equilibrium	minimum (t,y(t))	next equilibrium
A	$(?,?)$	$t = ?$	$(?,?)$	$t = ?$
B	$(?,?)$	$t = ?$	$(?,?)$	$t = ?$
C	$(?,?)$	$t = ?$	$(?,?)$	$t = ?$

3. All bets are off by the time $v = 5$. Explain what happens for A in particular the limiting value as $t \to \infty$. Compare to B.

4. All bets are off by the time $v = 5$. Explain what happens for D in particular the limiting value as $t \to \infty$. Compare to C.