Directions: Use only ONE side of each page, use ink and a staple.

This lab is divided into two pieces. One piece (#1 and #2) is about limit cycles which are stable periodic solutions and it has nice pictures. The second piece (#3 and #4) is about unstable numerical problems. These are problems which have nice analytic solutions but they cannot be found by numerical methods. This is a numerical cautionary tale.

We spiral into a limit cycle. The system in question is the non-linear system, the system of equations is from electronics, \( V \) is voltage and \( I \) is current. A typical \( \alpha \) value is \( \alpha = 0.5 \).

\[
\begin{align*}
\frac{dI}{dt} &= -I(I^2 - \alpha) - V \\
\frac{dV}{dt} &= I
\end{align*}
\]

Don’t forget to explain how you got your numbers from your technology. Also remember clarity and presentation.

1. Follow three initial values \((I_0, V_0)\) of \((2, 2), (-2, 2)\) and \((0.3, 0)\) to the limit cycle by plotting the curves in phase space.

2. What effect should increasing the value of \(\alpha\) have? Show the limit cycle for \(\alpha = 1.0, 1.5, 2.0\) and \(2.5\). Describe what changes as \(\alpha\) changes.

3. A numeric cautionary tale: Consider the IVP \(y'' - 9y = 0, y(0) = 1, y'(0) = -3\), solve this analytically and numerically. Why does it go wrong?

4. Consider the IVP \(y'' - y = 0, y(0) = 1, y'(0) = -1\), solve this analytically and numerically. Why does this one behave better?