Directions: Use only **ONE** side of each page, use ink and a staple.

This lab is divided into two pieces. One piece (#1 and #2) is about limit cycles which are stable periodic solutions and it has nice pictures. The second piece (# 3 and #4) is about unstable numerical problems. These are problems which have nice analytic solutions but they cannot be found by numerical methods. This is a numerical cautionary tale.

We spirial into a limit cycle. The system in question is the non-linear system, the system of equations is from electronics, V is voltage and I is current. A typical α value is $\alpha = 0.5$.

$$\frac{dI}{dt} = -I(I^2 - \alpha) - V$$
$$\frac{dV}{dt} = I$$

Don't forget to explain how you got your numbers from your technology. Also remember clarity and presentation.

1. Follow three inital values (I_0, V_0) of $(2, 2)$, $(-2, 2)$ and $(0.3, 0)$ to the limit cycle by ploting the curves in phase space.

2. What effect should increasing the value of α have? Show the limit cycle for $\alpha = 1.0$, 1.5, 2.0 and 2.5 Describe what changes as α changes.

3. A numeric cautionary tale: Consider the IVP $y'' - 9y = 0, y(0) = 1, y'(0) = -3$, solve this analytically and numerically. Why does it go wrong?

4. Consider the IVP $y'' - y = 0, y(0) = 1, y'(0) = -1$, solve this analytically and numerically. Why does this one behave better?