1. Fun with series
   (a) Show $1/(1-x)^2 = \sum_{n=0}^{\infty}(n+1)x^n$ by differentiating the series for $1/(1-x)$.
   (b) Show $1/(1-x)^2 = \sum_{n=0}^{\infty}(n+1)x^n$ by squaring the series for $1/(1-x)$.
   (c) Show $(1-x)^{-2} = \sum_{n=0}^{\infty}(n+1)x^n$ using the binomial series with $p=-2$

2. True or False and a brief reason why or why not.
   (a) $2\cdot 4\cdot 6\cdots(2n) = 2^n n!$
   (b) $1\cdot 3\cdot 5\cdots(2n-1) = \frac{(2n)!}{2^n n!}$
   (c) $\sum_0^{\infty} 2^n x^n = 1/(1-2x)$
   (d) $\sum_{n=3}^{\infty}(n+2)^2 a_n x^{n-2} = \sum_{n=1}^{\infty}(n+4)^2 a_{n+2}x^n$
   (e) At $x=1$, the error in using $x - x^3/3!$ to approximate $\sin x$ is less than 0.01.
   (f) At $x=-1$, the eror in using $1+x+x^2/2$ to approximate e^x is less than 1/5
   (g) As a rule of thumb the error in using $\sum_0^N a_n x^n$ to approximate $\sum_0^{\infty} a_n x^n$ at $x = x_1$ is about $a_{N+1}x_1^{N+1}$
   (h) $\arctan x = \int dx/(1+x^2) = \int \sum(-1)^n x^{2n}\,dx = \sum(-1)^n x^{2n+1}/(2n+1)$.
   (i) $\sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)$ has one as its radius of convergence
   (j) $\sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)$ converges at both endpoints ($x=1$ and $x=-1$) in its interval of convergence.

3. Short answers (Characteristics)
   (a) Find all r so that $\phi(t) = e^{rt}$ is a solution to $y'' + 8y' + 12y = 0$
   (b) Find all r so that $\phi(t) = x^r$ is a solution to $x^2y'' + 8xy' + 12y = 0$
   (c) Find all r so that $\phi(n) = r^n$ is a solution to $a_{n+2} + 8a_{n+1} + 12a_n = 0$
   (d) Show $y'' + 8y + 12y = 0$ can be re-written as the first order system

$$\begin{pmatrix} y1' \\ y2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -12 & -8 \end{pmatrix}\begin{pmatrix} y1 \\ y2 \end{pmatrix}$$

   (e) Find all λ so that

$$\det\begin{pmatrix} 0-\lambda & 1 \\ -12 & -8-\lambda \end{pmatrix} = 0$$