MAP 2302 Diff-E-Qs **Test 1** 20 May 2005

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. Fun loving Fred is spiking the fruit punch with rum. Initially the punch bowl has 100 cups of alcohol free fruit punch which is being consumed at 2 cups per minute. Fred is pouring rum which is 50% alcohol (by volume) also at 2 cups per minute into the punch bowl. The punch is well stirred at all times.
   (a) Write an IVP for $q(t)$, the amount of alcohol in the punch bowl (do NOT solve).
   (b) We change the problem slightly, instead of 2 cups per minute output, the fruit punch is consumed at 3 cups per minute. Write and SOLVE an IVP for $V(t)$, the volume of fluid in the punch bowl.
   (c) Write an IVP for $q(t)$, the amound of alcohol in the punch bowl, in the face of this faster consumption (do NOT solve).

2. Solve the IVP problems:

$$\text{(A) } y' = \frac{3x^2 - e^x}{2y - 5} \qquad y(0) = 1 \qquad\qquad \text{(B) } y' + 2y = te^{-2t} \qquad y(1) = 0$$

3. True or False and a brief reason why or why not.
   (a) The IVP $y' + x^2 y\sqrt{x} = \ln x,\ y(0) = 5$ is non-linear.
   (b) The ODE $(y')^5 + y''' - \sqrt{y''y} = \sin(\cos t)$ is fifth order.
   (c) The ODE $M(x, y) + N(x, y)dy/dx = 0$ is exact if $\partial N/\partial y = \partial M/\partial x$.
   (d) The function e^{2t} is a solution to $y' - e^{-4t}y^3 = e^{2t}$.
   (e) The function $\sin t/t$ is a solution to $ty' + y = \cos t$.
   (f) There is a first order ODE $y' = f(t, y)$ so that $y_1(t) = t,\ -5 < t < 5$ is the solution with the initial condition $y_1(0) = 0$ and $y_2(t) = 1 - t,\ -5 < t < 5$ is the solution with the initial contition $y_2(0) = 1$.
   (g) If $f(a) = 0$ and $f'(a) > 0$ then $y = a$ is a stable equilibrium solution to the autonomous $y' = f(y)$.
   (h) The IVP $y'(t) = y^{1/3}$, $y(0) = y_0$ has a unique solution if $y_0 \neq 0$.
   (i) Euler's method is usually better than Runge-Kutta.
   (j) If the step size is 1/10, the initial value $y(1) = 1$ and the ODE is $y' = 1 + y^2$, then the next point on the solution (according to Euler) is $y(1.1) = 3$.

4. The graph of $f(y)$ is pictured below. For the autonomous ODE $\frac{dy}{dt} = f(y)$, determine the critical (equilibrium) points, and classify each one as stable, unstable or semistable. Draw the phase line and sketch several graphs of solutions in the ty-plane. (At least one in each "region" of the phase line, and get the inflection points correct. (Yes there is a local min at G.))