MAP 2302 Diff-E-Qs **Test 2** 3 Jun 2005

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. For the inhomogenuous equations A–E complete a table like the one below. In the first column is the letter A–E, in the second column write the general solution to the associated homogenuous problem and in the third column give the correct "guess" for a particular solution using the method of undetermined coefficients. Do NOT solve.

Equation Letter	Gen Homo Solution	Undetermined Coeff "Guess"
A,B,C,D or E	?	?

$$\begin{aligned} y'' - 5y' + 6y &= \sin t && (A)\\ y'' - 2y' + 2y &= t^3 && (B)\\ y'' - 6y' + 9y &= e^{3t} && (C)\\ y'' + 2y' - 15y &= e^{3t} && (D)\\ y' &= e^{-2t}\sin 5t && (E) \end{aligned}$$

2. Use variations of parameters to find a particular solution to $y'' + p(t)y' + q(t)y = t^5$ if $y_1(t) = 1/t$ and $y_2(t) = t^3$ are solutions to the associated homogenuous equation.

3. True or False and a brief reason why or why not.

   (a) The ODE $y'' + \sqrt{x}y' - 3x^2e^xy = 3x^x$ is linear.

   (b) The characteristic equation of a second order linear ODE with constant coefficients always has real roots (sometimes equal, sometimes unequal).

   (c) If the Wronskian of $f(t)$ and $g(t)$ is non-zero at one point t_0, then the Wronskian is non-zero for every point t.

   (d) The identically zero function is always a solution to any linear homogenuous ODE.

   (e) If you multiply the mass m of an undamped spring-mass system by four, the the natural frequency ω is reduced by one half.

   (f) If you increase the γ of a critically damped spring-mass system, it becomes overdamped.

   (g) Resonance only happens with forced systems, not with free systems.

   (h) If $y_1(t)$ and $y_2(t)$ are both solutions to the linear ODE $L[y] = t^2 + 1$ then so is $y_1(t) + y_2(t)$.

   (i) If $y(t) = 5000e^{-2t}\sin(5t) + 10^6e^{-2t}\cos(5t) + 17\cos(50t)$ is the solution to a forced damped spring-mass system, then $17\cos(50t)$ is the transient solution.

   (j) The functions $\sin^2 t$ and $\cos^2 t$ are linearly independent.

4. A undamped spring-mass system uses a 2 kg mass and a spring which is elongated 1/8 m when a 1 N force is appled. The system is forced by $g(t) = (42/100)\sin(5t)$ N find the solution with initial values $y(0) = 0$ and $y'(0) = 1/10$.