Directions: Show ALL work for credit; Give EXACT answers when possible; Start each problem on a SEPARATE page; Use only ONE side of each page; Be neat; Leave margins on the left and top for the STAPLE; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded.

1. For the ODE \((x - \pi)^2(5 - x)y'' + \sin(x)y' + y/(1 + x^2) = 0\)
   
   (a) Find all singular points and determine if whether each is regular or irregular.
   
   (b) Determine a lower bound to the radius of convergence of the series solution about each given point 
   \(x_0: x_0 = 0, x_0 = 7, \) and \(x_0 = 4\)

2. Find the inverse Laplace transform of 
   \[
   \frac{7}{s^2 - 5s + 6} + 2e^{-\pi s} \frac{s + 5}{(s + 2)^2} + 3 - e^{-5s} \frac{1}{(s - 2)^4}
   \]

3. True or False and a brief reason why or why not.
   
   (a) The ODE \((x - \pi)^2(5 - x)y'' + \sin(x)y' + y/(1 + x^2) = 0\) is linear.
   
   (b) The radius of convergence of \(\sum_{n=0}^{\infty} 3^n x^n\) is 3.
   
   (c) If \(c > 0\), then \(u_c(t) = \int_0^t \delta(s - c) \, ds\).
   
   (d) \(\sum_{n=0}^{\infty} (ix)^n/n! = \sum_{n=0}^{\infty} (-1)^n x^{2n}/(2n)! + i \sum_{n=0}^{\infty} (-1)^n x^{2n+1}/(2n + 1)!\)
   
   (e) For constants \(a\) and \(b\) and for \(t \geq 0\) the solutions to the IVP \(y'' + ay' + by = 0, \ y(0) = 0, \ y'(0) = 1\) and the IVP \(y'' + ay' + by = \delta(t), \ y(0) = 0, y'(0) = 0\) are the same.
   
   (f) For all \(t, |t| = tu_0(t) - t\).
   
   (g) \(\sum_{n=0}^{\infty} (n^2 - n + 2) a_{n-3} x^{n+1} = \sum_{n=0}^{\infty} (n^2 - 5n + 8) a_{n-5} x^{n-1}\)
   
   (h) The function \(y(t) = u_5(t)(t - 5) - u_7(t)(t - 7)\) is continuous for every \(t\).
   
   (i) The general solution to \(x^2y'' + 2xy' + 2y = 0\) is \(C_1 x^{-1} \cos(\ln x) + C_2 x^{-1} \sin(\ln x)\)
   
   (j) If \(y_1(0) = 1, \ y'_1(0) = 0\) and \(y_2(0) = 0, \ y'_2(0) = 1\) and \(y_1(t)\) and \(y_2(t)\) are both solutions of \(y'' + y' + y = 0\), then their Laplace transforms satisfy \(Y_1(s) = sY_2(s)\).

4. Find the first five non-zero terms of the series solution about \(x_0 = 0\) to the IVP \(y'' - xy = 0, \ y(0) = 5, \ y'(0) = 7\).

Enjoy the rest of the summer!