MAP 2302 Diff-E-Qs **Test 3** 17 Jun 2005

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. For the ODE $(x-\pi)^2(5-x)y'' + \sin(x)y' + y/(1+x^2) = 0$
   (a) Find all singular points and determine if whether each is regular or irregular.
   (b) Determine a lower bound to the radius of convergence of the series solution about each given point x_0: $x_0 = 0$, $x_0 = 7$, and $x_0 = 4$
2. Find the inverse Laplace transform of
$$\frac{7}{s^2-5s+6} + 2e^{-\pi s}\frac{s+5}{(s+2)^2+3^2} - e^{-5s}\frac{1}{(s-2)^4}$$
3. True or False and a brief reason why or why not.
   (a) The ODE $(x-\pi)^2(5-x)y'' + \sin(x)y' + y/(1+x^2) = 0$ is linear.
   (b) The radius of convergence of $\sum_{n=0}^{\infty} 3^n x^n$ is 3.
   (c) If $c > 0$, then $u_c(t) = \int_0^t \delta(s-c)\,ds$.
   (d) $\sum_{n=0}^{\infty}(ix)^n/n! = \sum_{n=0}^{\infty}(-1)^n x^{2n}/(2n)! + i\sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)!$
   (e) For constants a and b and for $t \geq 0$ the solutions to the IVP $y'' + ay' + by = 0,\ y(0) = 0, y'(0) = 1$ and the IVP $y'' + ay' + by = \delta(t),\ y(0) = 0, y'(0) = 0$ are the same.
   (f) For all t, $|t| = tu_0(t) - t$.
   (g) $\sum_{n=7}^{\infty}(n^2-n+2)a_{n-3}x^{n+1} = \sum_{n=9}^{\infty}(n^2-5n+8)a_{n-5}x^{n-1}$
   (h) The function $y(t) = u_5(t)(t-5) - u_7(t)(t-7)$ is continuous for every t.
   (i) The general solution to $x^2y'' + 2xy' + 2y = 0$ is $C_1x^{-1}\cos(\ln x) + C_2x^{-1}\sin(\ln x)$
   (j) If $y_1(0) = 1,\ y_1'(0) = 0$ and $y_2(0) = 0,\ y_2'(0) = 1$ and $y_1(t)$ and $y_2(t)$ are both solutions of $y'' + y' + y = 0$, then their Laplace transforms satisfy $Y_1(s) = sY_2(s)$.
4. Find the first five non-zero terms of the series solution about $x_0 = 0$ to the IVP $y'' - xy = 0,\ y(0) = 5,\ y'(0) = 7$.

Enjoy the rest of the summer!