Directions: Show ALL work for credit; Give EXACT answers when possible; Start each problem on a SEPARATE page; Use only ONE side of each page; Be neat; Leave margins on the left and top for the STAPLE; Nothing written on this page will be graded;

1. Show $u = e^{-t}\sin 5x$ is a solution to $\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial u}{\partial t}$

2. Find the general solution to $y'' - 2y' + 5y = 0$

3. Compute and simplify the integral below assuming $n$ is a positive integer. Your answer should be trig-function-free. Hint: At the end, consider $n$ even and $n$ odd cases separately.

$$\int_0^\pi x \cos nx \, dx$$

4. A function $u(x, y)$ is also a function of the polar coordinate $r$ and $\theta$ via $u(r \cos \theta, r \sin \theta)$. Use the chain rule to show $\frac{\partial u}{\partial r} \cos \theta - \frac{1}{r} \frac{\partial u}{\partial \theta} \sin \theta = \frac{\partial u}{\partial x}$

Hint

5. True or False and a brief reason why or why not

Here $u(t)$ is the unit step function which is 0 for $t < 0$ and 1 otherwise.

(a) The function graphed below left is $u(t - 1) - u(t - 3)$

(b) The function graphed above right is $(t - 1)u(t - 1) - (t - 1)u(t - 2)$

(c) The trigonometric functions $\sin x$, $\cos x$, and $\tan x$ all have fundamental period $2\pi$.

(d) The following are trig identities $\sin 2x = 2 \cos x \sin x$ and $\cos 2x = \cos^2 x - \sin^2 x$

(e) The ODE $y'' + \sqrt{x}y' - 3x^2 e^x = 3x^3$ is linear.

(f) The ODE $(y')^2 + y'' + y^2 = \sin t \cos t$ is second order.

(g) $e^{2x} = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$

(h) The radius of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{2^n}$ is $\frac{1}{2}$

(i) The gradient of a function $f(x, y, z)$, $\text{grad } f = \nabla f$ is a scalar.

(j) The divergence of a function $f(x, y, z)$, $\text{div } f$ is a scalar.