A Simple Ecological Model

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Ecology

1. The branch of biology that deals with the relations of organisms to one another and to their physical surroundings.

2. The study of the interaction of people with their environment.

3. The political movement that seeks to protect the environment, especially from pollution.
The Undergraduate Math Majors:

- Pure Math
- Applied Math
- Actuarial Science
- BioMathematics
- FSU-Teach/Mathematics
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The FSU Mathematical Environment

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Why come to FSU for Mathematics?

- Access to Research as an Undergraduate
- Access to Graduate Classes
- Depth of the faculty and computer resources
- Upper level classes are small
Created with a Hughes Grant with the goal of teaching more math to biologists and perhaps more biology to mathematics students.

Uses the power of mathematical software (matlab, maple) to reduce the mathematical pre-requisite to only one semester of Calculus.

Why Mathematics is Biology’s next microscope.

Why Biology is Mathematics’s next Physics.
MAP 2480 Students

- Course is a 1-hour MAP 2480 Biocalculs Lab
- Biology majors, many Pre-Med, with a few biomath majors.
- Calculus I: MAC 2311 pre-requisite
- Some students are 3-4 years from Calculus
- Pre-meds work for grades, cram instead of learning
- No programming experience
Mathematical Modeling

Figure: The Modeling Process Cycle
Why are there so many different species?

We give evidence as to why this is a hard question, we don’t answer it.

In fact, for each kind of resource, there is only one species using that resource.

Diversity of species implies diversity of resources.
Teaser Question for this Ecology Lab

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Two more terms

- **Spacial**: of or relating to space.

- **Stochastic**: randomly determined; having a random probability distribution or pattern that may be analyzed statistically but may not be predicted precisely.

- **Curve Fitting**: the process of constructing a curve or mathematical function that has the best fit to a series of data points.
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Life on a checkerboard

- Plants grow on a checkerboard
- Each square has one plant
- Reproduction via nearest neighbors

Repeat:
- select plant
  - Voter Model, plant is replaced by a neighbor’s offspring
  - Invasion Process, plant’s offspring replaces a neighbor

generation is $nm$ births for $n \times m$ checkerboard.
The Colormap

<table>
<thead>
<tr>
<th>Species</th>
<th>Image Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>black</td>
</tr>
<tr>
<td>2</td>
<td>red</td>
</tr>
<tr>
<td>3</td>
<td>green</td>
</tr>
<tr>
<td>4</td>
<td>blue</td>
</tr>
<tr>
<td>5</td>
<td>yellow</td>
</tr>
<tr>
<td>6</td>
<td>magenta</td>
</tr>
<tr>
<td>7</td>
<td>cyan</td>
</tr>
</tbody>
</table>

The colormap installed by init.m; (careful 0 is also black).
n=10; A = zeros(n,n);
for i=1:n, for j=1:n,
A(i,j) = mod(i+j,4)+1;
end; end;
cm = [ 0 0 0; % 1 black
1 0 0; % 2 red
0 1 0; % 3 green
0 0 1; % 4 blue
1 1 0; % 5 yellow
1 0 1; % 6 magenta
0 1 1 ]; % 7 cyan
colormap(cm);
% 0 also gets black (why the plus one)
One generation later

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init;
image(A); % Scilab uses Matplot
rand(’seed’, 1234);
A = generationip(A);
ingame(A);

To get a movie:

init; for i=1:100;
A=generationip(A); image(A); pause(0.01);
end;
Experience with this model shows two behaviors:

1. The colors blotch together; plants of the same color tend to clump or cluster together.
2. Eventually all rectangles have the same color, one species wins and the others die out.
The second model

Suppose there were at most 2 species and we consider only the population of species 1. Each event in our old model did one of these things:

1. A plant was replaced by the same species. Population change: 0
2. A plant of species 1 was replaced by species 2. Population change: -1
3. A plant of species 2 was replaced by species 1. Population change: +1

And the last two events are equally likely. (Choose the edge and the then the direction.)
Dunkard’s walk on $2 \times 2$

![Diagram showing the transitions with probabilities](image)

$T = \begin{bmatrix}
1 & 0.5 & 0 & 0 & 0 \\
0 & 0 & 0.5 & 0 & 0 \\
0 & 0.5 & 0 & 0.5 & 0 \\
0 & 0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 1
\end{bmatrix}$

**Figure:** Diagram showing the transitions with probabilities

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If $X = \begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_n \end{bmatrix}$ with $x_i \geq 0$ and $\sum x_i = 1$, then as $n \to \infty$, $T^n X \to \begin{bmatrix} s & 0 & 0 & \ldots & 1 - s \end{bmatrix}$ some $s$, $0 \leq s \leq 1$. Fixed points of $T$, eigenvectors for the eigenvalue 1.
1. For loop output (for $i = a : b$, end;)
2. Next random number after seeding
3. Who is the winner after seeding
4. The movie – oracle question
5. Two steps for the dunk
6. 3 or 4 for the dunk
7. limit $T^n X$
8. Movie 2: 25x25 and red vs green – oracle question
Often we will have complex commands for you to do which are not easily graded on the computer. After you have done the task, as one of the instructors to check your work. If it is correct he will give you the answer to question “What is the answer to question number $a$?”

Answer: There is a sheet with the answers. The answer is the next random number for a simple-to-compute random number generator.
Monte Carlo simulations to estimate time to domination.
Geographic shapes (islands and land bridges) instead of checkerboards.
Other Ecological Labs

- Logistic Model of Population Growth
- Logistic Model of Population Growth plus Harvesting
- SIR epidemic model
- Predator Prey Model
Logistic Growth

Various sketches of $P$ versus $t$

Population ($P$) vs. time ($t$)

0 20 40 60 80
−200 0 200 400 600 800

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Logistic Growth with Harvesting

Harvesting 8 fish

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In a total population of $n$ individuals there are, at any time $t$,
- $x(t)$ members of the population who are susceptible to a contagious disease,
- $y(t)$ infectious carriers of this contagious disease,
- $z(t)$ individuals who are recovered and immune.
It is clear $x(t) + y(t) + z(t) = n$ for all $t \geq 0$.

\[
\begin{align*}
\frac{dx}{dt} &= -\lambda x(t)y(t) \\
\frac{dy}{dt} &= \lambda x(t)y(t) - \mu y(t) \\
\frac{dz}{dt} &= \mu y(t)
\end{align*}
\]
SIR Epidemic Model

SIR: S in blue, I in red, R in green; \( \lambda = 0.001, \mu = 1/14 \)

Days

People

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Predator Prey Model

Predator (Lynx), Prey (Hare) data from Web

- Snowshoe Hare
- Canada Lynx

Year

Pop (thousands)
Predator Prey Model

Lotka-Volterra: Predator Prey

- carrots
- rabbits

population

year
Stochastic and spatial models provide a more general first view of modeling as compared to curve fitting.

A nice animation to amuse the students.

A dunkards walk, monte carlo, islands, and more.
- Stochastic and spacial models provides a more general first view of modeling as compared to curve fitting.
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Summary

- Stochastic and spacial models provides a more general first view of modeling as compared to curve fitting.
- A nice animation to amuse the students.
- A dunkards walk, monte carlo, islands, and more.