Giving Data Formula-Like Properties

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Use to do Maple assignments in Calculus 3 but stopped.
Computer literacy has replaced programming.
Computer classrooms are hell, bored students are surfing.
Our Maple license didn’t include student copies.
Zebrahood: Failing students could blame Maple and by association the few professors that required it.
Why Scilab? Why not Calculus 3?

- Zebrahood: Use in all sections or in none.
- Free matlab clone selected from a list at a time when matlab licenses were too high. (Another professor (Steve Blumsack) selected Scilab over Octave and other clones.)
- Matlab is very popular and useful. Scilab is better in some ways and worse in others.
- Some Scilab classes: ODE, PDE for engineers, Matrix Algebra.
- Graph Theory, Scilab has nice graph theory package.
- Matlab in BioCalculus Lab.
Created with a Hughes grant joint with Biology. Has only a Calculus I co-requisite.

1-hour computer lab mets 75 minutes in a computer classroom.

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Hughes grant had money, 4 TAs for 96 students plus a professor to coordinate (4 sections of 24 students.)

The grant ran out. Moved to larger computer rooms (40 seats) and no Matlab.

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Ten zillion toolboxes, links with word. Derivatives, integrals, solving and evaluating functions using maple via the symbolic toolbox. Modelling is presented as curve fitting then doing calculus on the fitted curve and not dealing with the data directly. Motivation: replace this with direct data tools. And this turned out more enjoyable than I thought it would. Still haven’t tried it on students.
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\[ \text{data} = \begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_n \\ y_1 & y_2 & y_3 & \ldots & y_n \end{bmatrix} \]

\[ \text{datadiff} = \begin{bmatrix} \frac{x_1 + x_2}{2} & \frac{x_2 + x_3}{2} & \ldots & \frac{x_{n-1} + x_n}{2} \\ \frac{y_2 - y_1}{x_2 - x_1} & \frac{y_3 - y_2}{x_3 - x_2} & \ldots & \frac{y_n - y_{n-1}}{x_n - x_{n-1}} \end{bmatrix} \]

One less data point at the midpoints. Data could be the piecewise linear curve connecting the points. DataDiff could be piecewise constant with jumps.

Dividing \( 2\pi \) into 100 equal pieces, using the midpoint reduces the error on the derivative of \( \sin t \) from 0.0314056 to 0.000164.
function yprime = datadiff(w)
% assumes domain w(1,:) range w(2,:)
x = w(1,:); y = w(2,:);
midpts = (x(1:end-1)+x(2:end))/2;
slopes = diff(y) ./ diff(x);
yprime = [midpts ; slopes];

It is in a matlab m-file, named datadiff.m; Scilab requires $ instead of end, but doesn’t require the separate file. While % is the matlab comment character, // is the scilab comment character.
Lets look at a DataZero instead.

\[ data = \begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_n \\ y_1 & y_2 & y_3 & \ldots & y_n \end{bmatrix} \]

We want to find all points \( x_c \) where the piecewise linear curve connecting the points crosses the \( y \)-axis. So if \( y_i > 0 > y_{i+1} \) we would want \( x_c = ( -y_{i+1} x_i + y_i x_{i+1} ) / ( y_i - y_{i+1} ) \). Two other cases, direct hits (\( x_i \) when \( y_i \) is zero) and \( y_i < 0 < y_{i+1} \).
function roots = datazero(w)
% assumes domain w(1,: ) range w(2,:)
x = w(1,:); y = w(2,:); ay = abs(y);
jy = ay(1:end-1)+ay(2:end);
oscilate = y(1:end-1) .* y(2:end);
i = find(oscilate < 0);
g0 = (ay(i+1).*x(i)+ay(i).*x(i+1))./jy(i);
d0 = x(find(y==0));
roots = sort[d0 g0];

Vectorized code, there are no for loops. Handles multiple roots. If no roots, returns an empty array. Fails(?) for data with two consecutive zeros.
Given data below and a point $x_c$ with $x_1 < x_c < x_n$ what should be the value of $y_c$? Many possible answers. Two that we have already consisted

- Piecewise linear approximation.
- Nearest neighbor evaluation.
- General curve fitting also fits this model.
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function y0 = dataeval(x0, w)
% assumes domain w(1,:) range w(2,:)
x = w(1,:); y = w(2,:); xx = x - x0;
eval0 = y(find(x==0));
oscilate = xx(1:end-1) .* xx(2:end);
i = find(oscilate < 0);
eval2 = (xx(i+1).*y(i)-xx(i).*y(i+1))./(xx(i+1)-xx(i));
y0 = union[eval0 eval2];

This is not vector friendly, x0 needs to be a scalar.
function wint = dataint(w)
% assumes domain w(1,:) range w(2,:)
x = w(1,:); y = w(2,:); n = size(x,1);
newx = zeros(1,n+1); newy = zeros(1,n+1);
dx = x(2:end) - x(1:end);
newx(2:end-1) = (x(2:end)+x(1:end-1))/2;
newx(1) = newx(2)-dx(1);
newx(n+1) = newx(n)+dx(n-1);
newy(1) = 0;
for i = 1:n,
newy(i+1) = newy(i)+y(i)*dx(i);
end;
wint = [newx; newy];
Nearest Neighbor Evaluation

In the abstract, what all evaluations can do is given

\[
data = \begin{bmatrix} x_1 & x_2 & x_3 & \ldots & x_n \\ y_1 & y_2 & y_3 & \ldots & y_n \end{bmatrix}
\]

is to find functions \( f_i(x) \) based on the first row and evaluate according to

\[
\sum_{i=1}^{n} f_i(x) y_i
\]

the goal is to find vector friendly \( f_i \). For nearest neighbor, we want \( f_i \) to be the characteristic function of the interval \((m_{i-1}, m_i)\) where \( m_i \) is the midpoint of \( x_i \) and \( x_{i+1} \). (First approximation.)
The goal is vector friendly step functions. And it turns out the sign function works as a building block. Define \( \text{step} = (\text{sign} + 1)/2 \) so that?

\[
\text{step}(x) = \begin{cases} 
1 & x > 0 \\
\frac{1}{2} & x = 0 \\
0 & x < 0 
\end{cases}
\]

so that the resulting functions are vector friendly and make sense at the midpoints.

\[
f_i(x) = \text{step}(x - m_{i-1}) \text{step}(m_i - x)
\]
Summary

Working with data rather than curve fitting is doable and at least mildly interesting.

3D applications: scanner point clouds data are worked with directly. So there are applications.
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