8B – Doubling Time and Half-Life

The time required for each doubling in exponential growth is called the Doubling Time.
The time required to reduce by one half in exponential decay is called the Half-life.

**Doubling Time:**
Consider an initial amount of 10000 and a doubling time of 20 years:

In 20 years, or _____ doubling time, the population increases by a factor of ____ , to a new population of ____________.

In 40 years, or _____ doubling time, the population increases by a factor of ______ , to a new population of ____________.

In 60 years, or ______ doubling time, the population increases by a factor of ______ to a new population of ________________.

**Half-life:**
Consider an initial amount of 10000 and a half-life of 20 years:

In 20 years, or ____ half-life, the population decreases by a factor of ____ , to a new population of ________________.

In 40 years, or _____ half-lives, the population decreases by a factor of _________ , to a new population of ________________.

In 60 years, or ______ half-lives, the population decreases by a factor of ________ , to a new population of ________________.

**Generalizing in each case:**

After a time $t$, an exponentially growing quantity with a doubling time of $T_{Double}$ increases in size by a factor of $2^{t/T_{Double}}$. The new value of the growing quantity is related to its initial value (at $t=0$) by

new value = initial value * $2^{t/T_{Double}}$
After a time $t$, an exponentially decaying quantity with a half-life of $T_{\text{Half}}$ decreases in size by a factor of 

$\left(\frac{1}{2}\right)^{t/T_{\text{Half}}}$. The new value of the decaying quantity is related to its initial value (at $t=0$) by

\[
\text{new value} = \text{initial value} \times \left(\frac{1}{2}\right)^{t/T_{\text{Half}}}
\]

Examples:

1) (#28, pg 506) Prices are rising with a doubling time of 4 weeks. By what factor do prices rise in a year?

2) (#30, pg 507) The initial population of a town is 15,600, and it grows with a doubling time of 8 years. What will the population be in 12 years? In 24 years?

3) (#32, pg 507) The number of cells in a tumor doubles every 6 months. If the tumor begins with a single cell, how many cells will there be after 3 years? After 6 years?

4) (#42, pg 507) The half-life of a radioactive substance is 250 years. If you start with some amount of this substance, what fraction will remain in 70 years? In 140 years?
5) (#46, pg 507) The current population of an endangered animal species is 1 million, but it is declining with a half-life of 25 years. How many animals will be left in 40 years? In 70 years?

**Rule of 70 approximations**

**Doubling:**
For a quantity growing exponentially at a rate of $P\%$ per time period, the doubling time is *approximately*

$$T_{Double} \approx \frac{70}{P}$$

**Half-life:**
For a quantity decaying exponentially at a rate of $P\%$ per time period, the half-life is *approximately*

$$T_{Half} \approx \frac{70}{P}$$

*These approximations work best for small growth/decay rates and break down for rates over about 15%.*

6) A city’s population is growing at a rate of 2.0% per year. What is the approximate doubling time?

7) (#49, pg 508) Urban encroachment is causing the area of a forest to decline at a rate of 10% per year. What is the approximate half-life?
Be sure you can/know:
• The approximation formulas.
• How to use the “new-value” formulas. (They will be given on test.)