SAVING AND INVESTING

In this section we hope to impress upon you the importance of investing. By making monthly saving a priority now, you can greatly improve your financial status in the future. There are many ways to invest. We will not explore these options here but will instead focus on the power of financial planning.

Savings Plan Formula

In section 13.1 you learned the formula for finding the future value of money that is saved earning compound interest. The following formula gives the future value of a savings or investment account in which you make regular deposits.

> If **D** represents the amount of a regular deposit, **r** the annual interest rate expressed as a decimal, **m** the number of equal compounding period (in a year), and **t** the time in years, then the future value, **F**, of the account, is:
>
> $$F = \frac{D\left[\left(1+\frac{r}{m}\right)^{mt} - 1\right]}{\left(\frac{r}{m}\right)}.$$
>
> NOTE: The regular payments must be made on the same schedule as the compounding periods.

EXAMPLE 1 Suppose you deposit $100 per month in a savings account that earns 7.3% interest compounded monthly. How much money will you have in the account after 35 years? How much interest will you have earned?

Solution:

D = 100, r = 0.073, m = 12, and t = 35

Caution: When plugging into your calculator it is important either to use the parentheses exactly as they are written in the formula or make sure that you divide the answer to the numerator by the answer to the denominator.

Also, it is recommended that you calculate **mt** ahead of time to avoid an order of operations mistake.

mt = 12 × 35 = 420 (notice that this is the total number of $100 deposits you will have made)

Here is the calculation (^ represents the power function on a graphing calculator):

$F = 100 \times ((1 + 0.073 \div 12) ^\wedge 420 - 1) \div (0.073 \div 12) = \$193,513.30$

How much of this amount is the interest you have earned?

In 35 years of monthly payments you will have deposited $100 \times 420 = \$42,000$

So the interest earned is \$193,513.30 – 42,000 = \$151,513.30.

EXAMPLE 2
Suppose instead of depositing \$100 a month you deposit \$50 a paycheck. You are paid biweekly. And suppose the compounding periods matches the payment schedule. How much will you have earned at 7.3% interest after 35 years?

$D = 50$, $r = 0.073$, $mt = 26 \times 35 = 910$ (if you are paid biweekly then your money would go through 26 compounding periods per year)

$F = 50 \times ((1 + 0.073 \div 26) ^\wedge 910 - 1) \div (0.073 \div 26) = \$210,587.55$

You will have deposited $50 \times 910 = \$45,5000$.

So the interest you have earned is \$210587.56 – 45,500 = \$165,087.55.

Car Plan

One major expense for people is a car. Using the loan formula from section 13.2 and the savings plan formula from this section let us examine a plan that will help you pay for your cars with cash in the future.

CAR LOAN 1:

You borrow \$10,000 at a 7% interest rate on a 3 year loan in order to pay for a used car. The payment is \$308.77 (verify this). You keep the car for 5 years but after paying off the 3-year loan you deposit \$308.77 into a savings account for the 2 years.

Calculate the future value of the deposits if you earn 4% compounded monthly. This amount is \$7701.62.

CAR LOAN 2:
You sell the car and buy another used car for \$12,000 by using your down payment of \$7700. Your loan this time will be \$12,000 – 7700 = \$4300. This time you take out a 1 year loan at 7% expecting to keep the car 5 years. Your payment this time is \$372.07 (verify this).After you pay back the loan you continue to pay the same amount deposited into a savings account for the next 4 years. If you can earn 4% interest compounded monthly, how much will you have for a down payment on your next car?
(answer: \$19,332.61)

Planning for the Future

Suppose you determine that you need $2 million dollars in a retirement account in order to retire comfortably after 45 years. How much should you deposit monthly in an account that pays 8% compounded monthly in order to achieve this goal?

Solution:

Using F = 2,000,000, r = 0.08, mt = 12 × 45 = 540 and plugging into the savings plan formula we have

$$2{,}000{,}000 = \frac{D\left(\left(1+\frac{0.08}{12}\right)^{540}-1\right)}{\left(\frac{0.08}{12}\right)}$$

2,000,000 = D(5274.539892) Note: It is important not to round until the end.
So, D = 2,000,000 ÷ 5274.539892 = $379.18.

EXERCISES

For each of the following find F, the future value of the investments. Then find the interest earned.

1) Regular yearly deposits of $10,000 for 30 years at 6.5% interest compounded yearly.
2) A) Regular quarterly deposits of $2500 for 30 years at 6.5% interest compounded quarterly.
   B) Regular quarterly deposits of $2500 for 30 years at 5.5% interest compounded quarterly.
3) Regular monthly deposits of $450 for 20 years at 7.25% interest compounded monthly.
4) Regular monthly deposits of $150 for 45 years at 7.25% interest compounded monthly.
5) Biweekly deposits of $75 made over a 45 year period at 7.25% interest compounded biweekly.

For #6 and #7 of the following determine the monthly deposit, D, necessary to achieve the given future value, F.

6) F = $200,000, r = 5.5% compounded monthly for 15 years
7) F = $1,500,000, r = 6.75% compounded monthly for 40 years
8) Suppose you have deposited $500 per month for 30 years. At the end of the 30 years the account holds $687,366.43. How much interest have you earned?

Challenge: Suppose you decide to double your monthly contributions to a retirement account every 10 years. Assume you can obtain a constant 8.3% interest rate compounded monthly. How much will be in the account after 40 years if you start by depositing $50 per month during the first decade? Hint: You will need to use the formula for compound interest from section 13.1 in addition to using the formula for regular deposits.

2nd Challenge: How much would you have if you followed this plan in reverse (i.e. contribute $400/month for the 1st decade, $200/month for the 2nde decade, etc.)? Note that you can use some of your work from the 1st challenge.

Answers:

1) $863,748.64, interest = $563,748.64
2) A) $910,673.48, interest = $610,673.48 B)$754,323.06, interest = $454,323.06
3) $241,663.52, interest = $133,663.52
4) $617,205.24, interest = $536,205.24

NOTE: Compare #1 and #2A to see how more frequent compounding affects investments. Compare #2A and #2B to see how great a difference 1% interest more makes. Compare #3 and #4 to see what a difference it makes to begin saving early in life (the person investing in #3 could be someone around 45 years old and the person in #4 could be someone about 20 years old)

5) $672,318.15, interest = $584,568.15
6) $717.50
7) $612.85
8) $507,366.43

Challenge:
For the 1st decade D = 50, r = 0.083, mt = 12 × 10 = 120 so F_1 = $9301.99. Over the next 30 years this will grow to $9301.99(1 + 0.083/12)^{(12 \times 30)}$ = $111,236.28

For the next decade D = 100. So F_2 = $18,603.98. Over the next 20 years this will grow to $18{,}603.98(1 + 0.083/12)^{(12 \times 20)}$ = $97,286.58.

In the 3rd decade D = 200 and you find F_3 = $37,207.97 which will grow to $85,086.27 by the end of the 40 years.

In the last decade D = 400 and so F_4 = $74,415.93.

Thus, at the end of 40 years you would have $111,236.28 + $97,286.58 + $85,086.27 + $74,415.93 = $368,025.06.

Answer to 2nd challenge: $1,136,308.71